Energetics, power and gross efficiency in cross country skiing

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January 27, 2026

Abstract

In contrast to running, cross-country skiing is characterized by the presence of non-zero external mechanical power output even on level terrain, arising primarily from ski–snow friction and aerodynamic drag. A quantitative description of the braking forces is essential for understanding the energetic demands and efficiency of skiing across a wide range of inclines and speeds.

In this work, we present a hybrid first-principles framework for modeling external mechanical power, metabolic energy expenditure, and gross efficiency in cross-country skiing. Although skiing involves complex, multi-limb locomotion, propulsion in specific techniques such as double poling can be associated with a well-defined drivetrain, enabling the formulation of measurable propulsive power despite technique-dependent variations in gross efficiency. The model introduces an incremental incline efficiency to distinguish between level and graded locomotion, and incorporates internal dissipation through a skiing economy parameter, yielding closed-form expressions for oxygen uptake, power output, and efficiency as functions of speed, grade, and surface conditions.

The framework is validated against a range of published treadmill and roller-ski studies, demonstrating good agreement with empirical observations of gross efficiency and oxygen consumption across moderate to steep inclines. For practical applications, simplified tables are provided to facilitate rapid estimation of speed, power, and metabolic demand under typical training and competition conditions.

1 Introduction

Since the dawn of exercise physiology as a scientific field, work rate has been extensively studied [1], generally expressed by means of oxygen consumption but also compared to mechanical work rate. However, the latter manifests very differently depending on the nature of the motion. In cycling, for instance, the mechanical work can be directly measured in the drivetrain, e.g. at the crank. It can further be shown that this measure agrees well with external restraining forces from rolling resistance, air resistance, and gravity, and that it is proportional to metabolic rate by means of a fairly consistent gross efficiency (GE).

In running, on the other hand, it is far more difficult to define the external work rate, as there is no well-defined drivetrain. Furthermore, the ratio between external power, as determined by motion against restraining forces, and the metabolic rate is far from constant; in other words, GE varies. This difference is the reason why mechanical power is a meaningful performance metric in cycling but not in running. Proxy-based wearables claiming to measure running power may be useful for intensity control, but not for performance analysis [2]. This limitation is not primarily due to insufficient measurement accuracy, but rather to ambiguity in the definition of power itself.

Having covered cycling and running as two extreme cases, we now turn to skiing, which lies somewhere in between. In skiing, a defined mechanical work is always present through motion against restraining glide friction. With increasing incline and speed, motion against gravity and air resistance contributes in a manner similar to cycling. Despite these similarities, the situation is more complex than in cycling for two important reasons: (i) gross efficiency varies with speed,

incline, and technique; and (ii) the drivetrain for measuring external forces is not as well defined as in cycling, although better defined than in running.

These two fundamental factors make the measurement and analysis of ski power more challenging than cycling power. However, in contrast to running power, which is ill-defined by definition, skiing power is a physically meaningful quantity and can be used for both intensity control and performance analysis provided that the measurement and the GE variations can be well-defined.

2 Power and energetics in cross-country skiing

To maintain a constant forward speed, the propulsive forces generated by the skier must balance the restraining forces arising from gravity, ski–snow friction, and aerodynamic drag. The total resisting force may be expressed as

$$F_r = mg\mu\cos\theta + mg\sin\theta + \frac{\rho C_d A}{2}u^2, \tag{1}$$

where μ is the coefficient of ski–snow friction, θ is the inclination angle, C_dA denotes the effective aerodynamic drag area, u is the skiing speed in $\mathrm{m\,s^{-1}}$, and ρ is the air density. The corresponding external mechanical power required to maintain constant speed is obtained by multiplication with speed,

$$P_r = F_r\,u = mg\mu u\cos\theta + mgu\sin\theta + \frac{\rho C_d A}{2}u^3. \tag{2}$$

Normalized by body mass, (2) can be written as

$$p_r = g\mu u\cos\theta + gu\sin\theta + \frac{\rho C_d A}{2m}u^3 \approx g(\mu + \mathrm{grade})u + \frac{\rho C_d A}{2m}u^3, \tag{3}$$

where the approximation assumes small angles, $\cos\theta \approx 1$ and $\sin\theta \approx \mathrm{grade}$.

Equation (3) highlights a key distinction between cross-country skiing and running: even on level terrain ($\theta = 0$), skiing involves a non-zero external mechanical power demand due to glide friction. In contrast, the net external mechanical power in steady-state level running vanishes. The purpose of this section is to establish a framework for modeling metabolic energy expenditure, external mechanical power, and the associated gross efficiency (GE) in cross-country skiing.

In direct analogy with running [3], we distinguish between the efficiency associated with level locomotion and the efficiency associated with graded locomotion. The latter is represented by an incline-referred incremental efficiency η_I, while the former is converted to metabolic power by means of a *quasi-gross efficiency* η^*.

2.1 A quasi-first-principles metabolic model for cross-country skiing

Energy expenditure during level-terrain skiing can be decomposed into two primary contributions: (i) the true mechanical work performed against glide friction μ, and (ii) the metabolic cost associated with internal dissipation due to cyclic acceleration and deceleration of the limbs. The latter is represented by an equivalent resistance μ_{eq}, or equivalently by the skiing economy SE. Importantly, μ_{eq} is not a physical snow-friction coefficient; rather, it is a lumped parameter that captures internal dissipation as an effective horizontal resistive work rate $mg\mu_{eq}u$, in direct analogy with the equivalent resistance introduced for level running [3].

Despite their different physical origins, both μ and μ_{eq} contribute to continuous non-conservative external power demands and are therefore converted to metabolic power through the same effective (quasi-gross) efficiency η^*. The additional energetic cost associated with positive vertical work during uphill locomotion is converted through the incremental incline efficiency η_I.

Neglecting aerodynamic drag for the moment, the mass-normalized metabolic power may be expressed as

$$\begin{aligned} p_{\text{met}} &= \frac{e_{O_2}}{60}\,\text{RMR} + gu\left(\frac{\mu\cos\theta + \mu_{eq}}{\eta^*} + \frac{\sin\theta}{\eta_I}\right) \\ &= \frac{e_{O_2}}{60}\,\text{RMR} + \frac{SE\,e_{O_2}}{1000}\,u + \frac{g\mu\cos\theta}{\eta^*}\,u + \frac{g\sin\theta}{\eta_I}\,u, \end{aligned} \tag{4}$$

where SE denotes the *skiing economy*, expressed in $\text{mL}\,\text{kg}^{-1}\,\text{km}^{-1}$. The second line follows by defining SE to capture the baseline internal dissipation term,

$$SE = 1000\frac{g\mu_{eq}}{\eta^* e_{O_2}}. \tag{5}$$

This definition is in direct analogy with the running economy used for level running. While the concept of skiing economy has appeared in previous studies [4, 5], it is less well established than its running counterpart. In the present framework, SE is associated exclusively with internal dissipative processes, whereas previous treatments also include glide friction within the economy term. The explicit exclusion of glide friction from SE is essential in order to obtain an interpretable expression for modeling energetics and gross efficiency in cross-country skiing. If μ were mixed into SE, it would vary with skiing conditions.

This formulation may alternatively be expressed in terms of oxygen uptake. The mass-normalized metabolic power can be mapped to an equivalent oxygen consumption rate,

$$VO_2 = \frac{60}{e_{O_2}}\,p_{\text{met}} \quad = \text{RMR} + SE\,u + \frac{60}{e_{O_2}}\,gu\left(\frac{\mu\cos\theta}{\eta^*} + \frac{\sin\theta}{\eta_I}\right). \tag{6}$$

2.2 Efficiency relations in cross-country skiing

As evident from the above section and furhter verified in empirical studies, gross efficiency in cross-country skiing decreases with decreasing incline but remains finite, unlike in running where gross efficiency approaches zero on level terrain. This behavior is naturally captured by representing internal dissipation through the equivalent resistance μ_{eq}, or equivalently through the skiing economy SE.

Based on the definition of gross efficiency as the ratio between external mechanical power output and metabolic power input, we define the gross efficiency in cross-country skiing as

$$\eta_{\text{gross}} = \frac{gu\,(\mu\cos\theta + \sin\theta)}{p_{\text{met}}} = \frac{gu\,(\mu\cos\theta + \sin\theta)}{\frac{e_{O_2}}{60}\,\text{RMR} + gu\left(\frac{\mu\cos\theta + \mu_{eq}}{\eta^*} + \frac{\sin\theta}{\eta_I}\right)}, \tag{7}$$

As the incline increases, the gross efficiency $\eta_{\text{gross}}(\theta)$ approaches a plateau, reflecting the fact that the conversion between mechanical and metabolic power cannot improve indefinitely. We therefore introduce a characteristic incline θ_I, referred to as the efficiency saturation angle, at which the gross efficiency reaches its saturated value $\eta_{G,\text{sat}}$.

Beyond this point, further increases in slope no longer result in an increase in effective gross efficiency. Accordingly, the effective (quasi-gross) efficiency η^* used to convert non-conservative external power demands is identified with the saturated gross efficiency, $\eta^* = \eta_{G,\text{sat}}$. Imposing the saturation condition $\eta_{\text{gross}}(\theta_I) = \eta_{G,\text{sat}}$ in (7) yields

$$\eta_{G,\text{sat}} = \frac{gu\,(\sin\theta_I - \mu_{eq})}{\frac{e_{O_2}}{60}\,\text{RMR} + \frac{gu\sin\theta_I}{\eta_I}}. \tag{8}$$

Notably, (8) has the same functional form as the corresponding expression obtained for running. This follows from treating glide friction as an external, non-conservative resistive power demand that is converted to metabolic power through the same saturated gross efficiency. Consequently,

under the saturation condition $\eta_{\text{gross}}(\theta_I) = \eta_{G,\text{sat}}$ and the identification $\eta^* = \eta_{G,\text{sat}}$, the Coulomb friction term cancels algebraically from the final expression.

Although mathematically motivated above, it may still seem counter-intuitive that the glide friction coefficient μ vanishes from (8). A physical interpretation is obtained by considering the limiting case $\mu \to 0$. In this case, the saturated gross efficiency $\eta_{G,\text{sat}}$ remains unchanged, while the characteristic incline θ_I at which saturation is attained is shifted. Thus, glide friction does not determine the level of the efficiency plateau, but rather the incline at which the system transitions into the saturation regime.

If $\frac{e_{O_2}}{60}\,\text{RMR} \ll \frac{gu\sin\theta_I}{\eta_I}$, (8) may be simplified to

$$\eta_{G,\text{sat}} \approx \eta_I \left(1 - \frac{\mu_{eq}}{\sin\theta_I}\right) \approx \eta_I \left(1 - \frac{\mu_{eq}}{\text{grade}}\right), \tag{9}$$

where the final approximation follows from $\sin\theta_I \approx \text{grade}$.

Alternatively, the internal losses may be expressed in terms of skiing economy rather than the equivalent resistance μ_{eq}. Retaining RMR and evaluating the expression along a $VO_{2,\max}$-limited trajectory yields

$$\eta_{G,\text{sat}}(\theta_I;\eta_I) = \frac{g\sin\theta_I\ (VO_{2,\max} - \text{RMR})}{VO_{2,\max}\left(\dfrac{SE\,e_{O_2}}{1000} + \dfrac{g\sin\theta_I}{\eta_I}\right)}, \tag{10}$$

where the skiing economy SE represents the metabolic cost associated with internal dissipation and limb motion. The glide friction coefficient μ does not appear explicitly in (10), as it contributes to both the external mechanical power and its metabolic translation through the same saturated gross efficiency, and therefore affects the location of θ_I rather than the level of $\eta_{G,\text{sat}}$.

Neglecting $\text{RMR} \ll VO_{2,\max}$ further yields

$$\eta_{G,\text{sat}}(\theta_I;\eta_I) = \frac{g\sin\theta_I}{\dfrac{e_{O_2}SE}{1000} + \dfrac{g\sin\theta_I}{\eta_I}} = \frac{1}{\dfrac{e_{O_2}SE}{1000\,g\,\sin\theta_I} + \dfrac{1}{\eta_I}}. \tag{11}$$

By substitution of SE in terms of μ_{eq}, (11) reduces to (9). Since $\sin\theta_I$ is bounded, it follows that the saturated gross efficiency $\eta_{G,\text{sat}}$ is bounded as well. In practically relevant cross-country skiing, sustained uphill locomotion rarely exceeds grades on the order of 20–30%, as steeper slopes are typically not maintainable due to technique- and force-related limitations. Experimental treadmill and roller-ski studies further indicate that gross efficiency increases from level to moderately steep terrain, but reaches a plateau within this range of inclines, with no evidence for continued improvement at higher grades [6, 7, 8, 9].

Consistent with these observations, we evaluate (11) for $\sin\theta_I \approx 0.2$–0.3, which yields saturated gross efficiencies in the range $\eta_{G,\text{sat}} \approx 22$–$28\%$ for experimentally reported values of the incremental incline efficiency $0.2 < \eta_I < 0.4$. For illustration, we adopt the representative value $\sin\theta_I \approx 0.3$ (corresponding to $\theta_I \approx 17°$). The resulting range of saturated gross efficiencies is in good agreement with classical observations of gross mechanical efficiency in steep uphill locomotion, which cluster around approximately 20–25%.

In the practically relevant regime where $\text{RMR} \ll VO_{2,\max}$, and for typical uphill skiing conditions where the grade rarely exceeds approximately 30%, we adopt the approximations $\sin\theta \approx \gamma$ and $\cos\theta \approx 1$. Under these assumptions, the quasi-gross efficiency may be written as

$$\eta_{\text{gross}}(\gamma;\eta_I) = \frac{g(\mu+\gamma)}{\dfrac{e_{O_2}SE}{1000} + \dfrac{g(\mu+\gamma)}{\eta_I}} = \frac{1}{\dfrac{e_{O_2}SE}{1000\,g(\mu+\gamma)} + \dfrac{1}{\eta_I}}. \tag{12}$$

Figure 1 illustrates the relationship between gross efficiency and incremental incline efficiency within the experimentally relevant range. The analysis demonstrates that the saturated gross

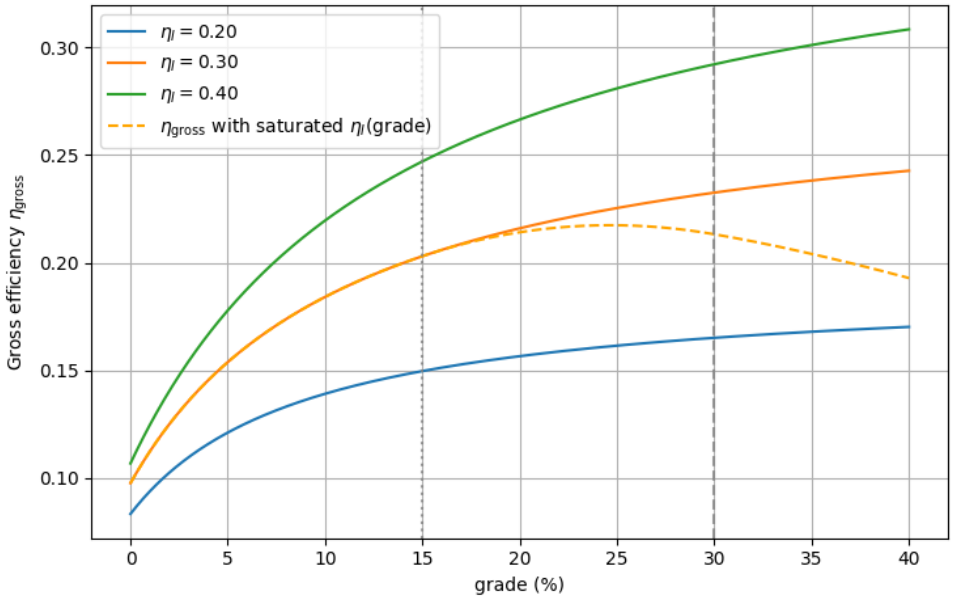


Figure 1: Gross efficiency as a function of incremental incline efficiency.

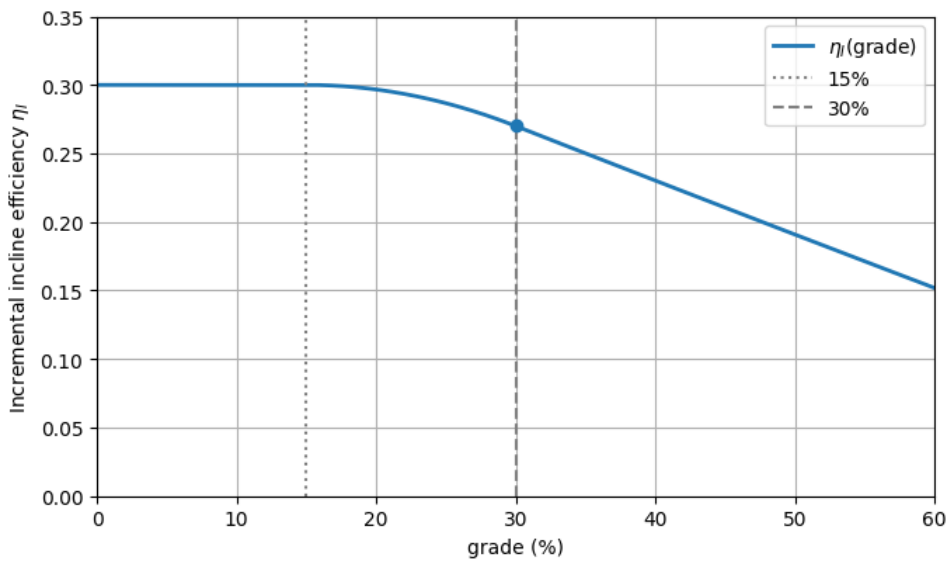


Figure 2: Illustrative decline of incremental incline efficiency with increasing grade.

efficiency remains strictly lower than the corresponding incremental incline efficiency. This bound arises from internal dissipation, represented here by the skiing economy SE, or equivalently by the effective resistance μ_{eq}.

The solid curves in Fig. 1 are obtained directly from (11) under the assumption that the incremental incline efficiency η_I can be treated as locally constant in the vicinity of the efficiency saturation angle θ_I. This approximation requires only that η_I varies sufficiently slowly near θ_I to be regarded as approximately constant, rather than remaining constant over the full range of inclines.

To account for the experimentally observed emergence of a plateau in gross efficiency at steeper grades, we introduce an explicit grade dependence of the incremental incline efficiency,

$$\eta_I(\gamma) = \frac{\eta_{I0}}{1 + \left(\dfrac{\gamma}{\gamma_0}\right)^p}, \tag{13}$$

where $\gamma \equiv \tan\theta$ denotes the treadmill grade expressed as a fraction (e.g., $\gamma = 0.30$ corresponds to 30%), η_{I0} represents the low-grade efficiency plateau, γ_0 sets the characteristic grade scale at which efficiency begins to decline appreciably, and p controls the sharpness of this decline. This functional form provides a smooth, monotonically decreasing description that captures the essential behavior that η_I remains close to its low-grade value at moderate inclines and decreases progressively as steeper grades introduce force- and technique-related limitations, as illustrated in Fig. 2.

Substituting (13) into (12) yields an explicit expression for the gross efficiency as a function of grade,

$$\eta_{\mathrm{gross}}(\gamma) = \frac{\eta_{I0}}{1 + \left(\dfrac{\gamma}{\gamma_0}\right)^p + \dfrac{\eta_{I0}\, e_{O_2} SE}{1000\, g(\mu + \gamma)}}, \tag{14}$$

which forms the basis for the numerical examples presented below.

Before proceeding to validation against published data, we utilize the inferred efficiency relations to formulate an explicit expression for oxygen consumption. Under the small-angle approximation and the identification of the quasi-gross efficiency with its saturated value, the mass-normalized oxygen uptake may be written as

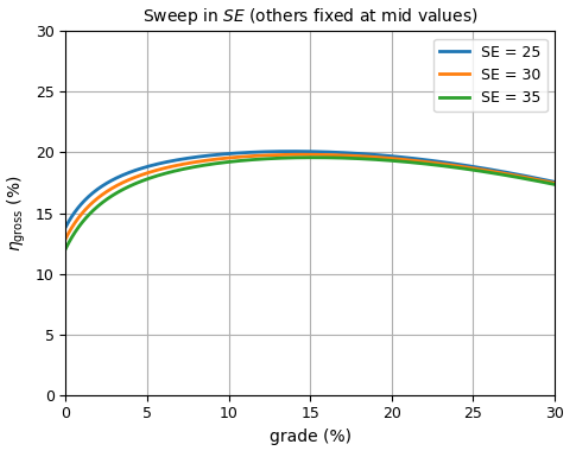
$$VO_2 \approx \mathrm{RMR} + \frac{60u}{1000} SE \left(1 + \frac{\mu}{\sin\theta_I}\right) + \frac{60}{e_{O_2}} gu \frac{\mu + \gamma}{\eta_I}, \tag{15}$$

where $\gamma \equiv \tan\theta \approx \sin\theta$ denotes the treadmill grade expressed as a fraction, and we have introduced the saturation grade $\gamma_I \equiv \tan\theta_I \approx \sin\theta_I$. Note that in (13), γ_0 denotes the half-value point of the decline in η_I, i.e., $\eta_I(\gamma_0) = \eta_{I0}/2$, whereas γ_I corresponds to the onset of the gross-efficiency plateau and therefore occurs at grades well below γ_0. For $p = 3$, this onset is approximated as $\gamma_I \approx \gamma_0/3$ (equivalently, $\gamma_0 \approx 3\gamma_I$). Note further that the coupling of the physical friction coefficient μ to the skiing economy term in (15) arises from identifying the quasi-gross efficiency with its saturated value, which algebraically links internal dissipation to the saturation grade.

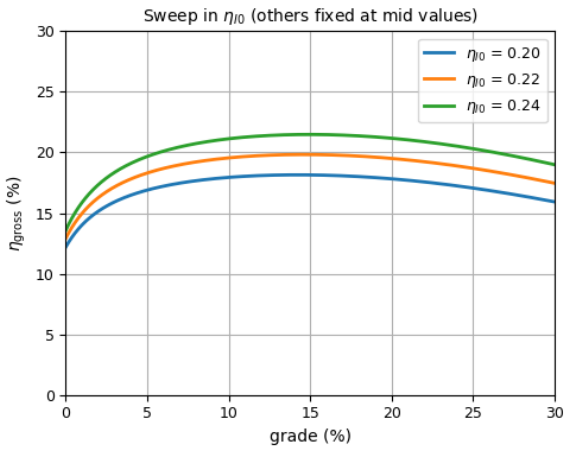
Introducing the speed variable v expressed in $\mathrm{km\,h^{-1}}$, with the corresponding relation $u = v/3.6$ for the conversion to $\mathrm{m\,s^{-1}}$, the expression in (15) can be written in the compact form

$$VO_2(\gamma) \approx \mathrm{RMR} + v\left[\frac{1}{60} SE \left(1 + \frac{\mu}{\gamma_I}\right)\right] + \frac{60g}{e_{O_2}\eta_{I0}} \left(\frac{v}{3.6}\right)(\mu + \gamma)\left(1 + \left(\frac{\gamma}{3\gamma_I}\right)^p\right), \tag{16}$$

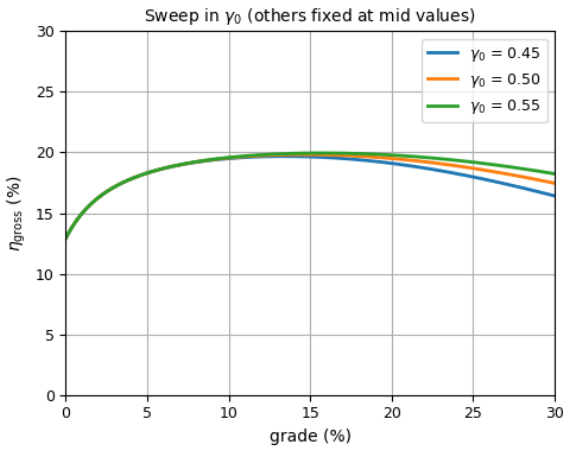
which provides a closed-form representation of oxygen uptake as a function of grade, internal dissipation, and technique-dependent efficiency parameters. Equation (16) is linear in the speed variable v, which makes it straightforward to solve explicitly for velocity as a function of $\dot{V}O_2$.



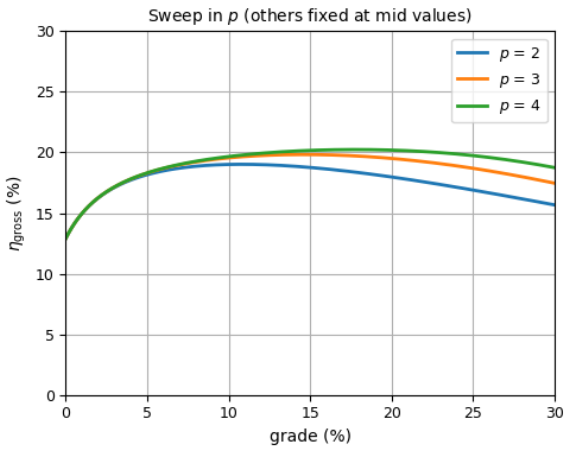
(a) Variation of skiing economy SE.



(b) Variation of low-grade incremental efficiency η_{I0}.



(c) Variation of characteristic grade scale γ_0.



(d) Variation of shape exponent p.

Figure 3: Gross efficiency η_{gross} as a function of grade for the analytic model (14). In each panel, one parameter is varied while the remaining parameters are fixed at representative mid-range values.

$$v(\gamma) \approx \frac{VO_2(\gamma) - \text{RMR}}{\frac{1}{60}\,SE\left(1+\frac{\mu}{\gamma_I}\right) + \frac{60g}{e_{O_2}\eta_{I0}}\left(\frac{1}{3.6}\right)(\mu+\gamma)\left(1+\left(\frac{\gamma}{3\gamma_I}\right)^p\right)}. \tag{17}$$

For simple practical use, Table 1 presents attainable speed for different VO_2 and inclinations.

2.3 Validation and tuning versus empirical data

In Section 2, an expression was derived for modeling energetics and gross efficiency in cross-country skiing as a function of grade. The full expression is presented in (14) and depends on four physically interpretable parameters: SE, η_{I0}, γ_0, and μ. The sensitivity to the model parameters was illustrated in Fig. 3, where it can be seen that the most critical parameters are SE and $\eta_I 0$, with the latter becoming increasingly important as grade increases.

The parameters γ_0 and p, which control the grade dependence of η_I, have a small influence for grades below 10%, but affect the results for grades beyond $0.5\gamma_0$. Reasonable values of SE and the baseline efficiency η_{I0} were selected based on empirical data in [10] and the requirement that SE should be invariant with incline. This results in $SE = 30$ for $\eta_{I0} = 22\%$, with $\gamma_0 = 0.5$

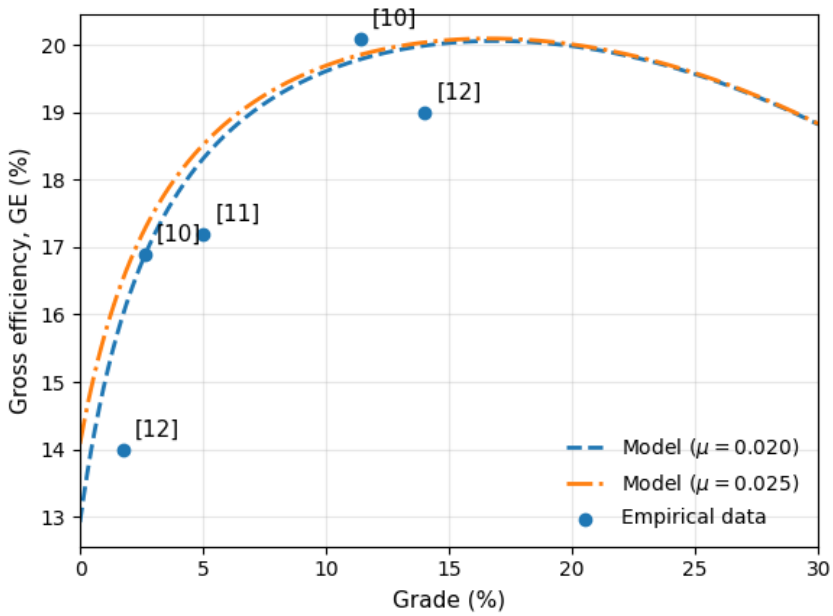


Figure 4: Modelled gross efficiency (GE) according to (14) compared to data from empirical studies.

and $p = 3$. Variations in γ_0 and p have a marginal influence for grades up to 15%, which is the range considered in the validation study.

Further validation against empirical data is illustrated in Fig. 4, showing modeled gross efficiency (GE) for two different values of rolling friction, covering the majority of available empirical studies and compared to empirical values of GE determined in [11] and [12]. Notably, [11] and [12] report similar trends as [10] but different absolute values, which may be explained by variations in measured VO_2 as different systems are used in the different studies.

At this point it should be emphasized that empirical GE values does vary with skiing technique. At steep inclines diagonal stride (DS) is more economic than double poling (DP), while at low inclines the opposite applies. It has been shown that there is critical incline where GE of DS exceeds GE of DP. In this framework, the presented GE model may be interpreted as highest possible GE assuming optimal technique selection, but it is also possible to tune the model parameter values for specific techniques, .e.g, DP, DS, or even free style.

3 Measuring power in cross-country skiing

To maintain a constant forward velocity, the resistive forces described in Section 2 must be balanced by propulsive forces generated by the skier. This section addresses the measurement of these propulsive forces and the associated mechanical power output in cross-country skiing.

3.1 The drivetrain of cross-country skiing

Similar to running, cross-country skiing is a complex mode of locomotion involving coordinated motion of both lower- and upper-body segments, which complicates the definition of a distinct drivetrain for mechanical power measurements. However, for a specific classical sub-technique, namely double poling, the drivetrain is well defined: propulsion is transmitted exclusively through the poles, and the external mechanical work can be directly associated with the axial pole force and its temporal variations

$$F_{\text{prop}}(t) = F(t) \times \cos\alpha(t), \tag{18}$$

where $F(t)$ is the momentaneous axial pole force and $\alpha(t)$ is the momentaneous angle between pole and ground. Figure 5 illustrates how the pole force varies during a poling cycle.

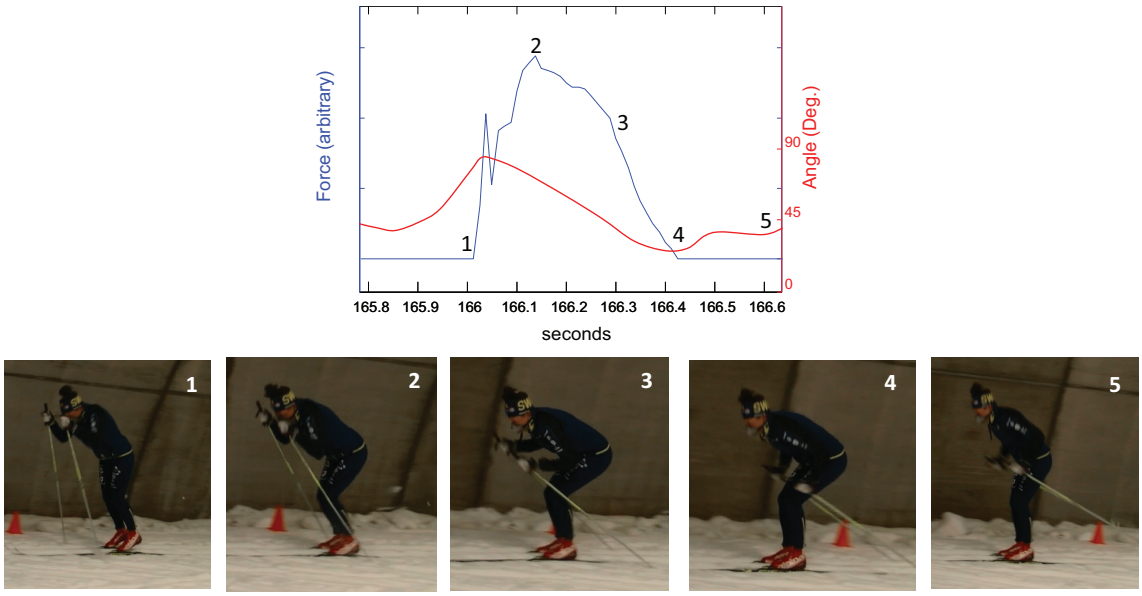


Figure 5: Axial pole force curve and its connection to different phases in the poling cycle.

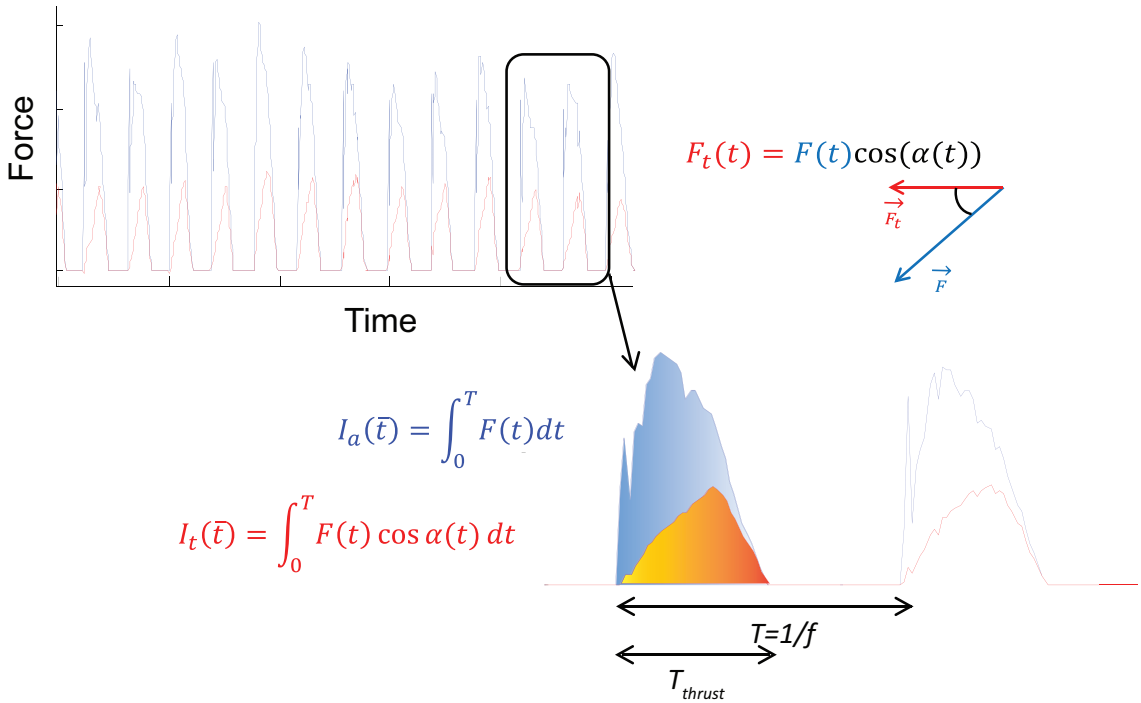


Figure 6: Consequtive polings broken down into impulse and frequency used to calculate propulsive force.

For deeper understanding, the propulsion from a single poling may be increased either by increasing the axial pole force or by increasing the fraction of the force that goes in the direction of motion by a more favourable temporal variation of angle and force. Mathematically, the propulsion from a single poling can be described by the impulse, i.e., the force integrated over

one poling cycle,

$$I_{\text{prop}} = \int_0^T F(t) \times \cos\alpha(t)dt = \int_0^{T_0} F(t) \times \cos\alpha(t)dt. \tag{19}$$

where T is the poling cycle time, counted from the start of one poling to the start of the next, including both pole thrust-duration and pole flight time. T_0 is the absolute thrust duration, i.e., the time the poles are in contact with ground.

At the end it is not the propulsion from a single poling that is of interest but the resulting propulsive force F_r, which also depends on the number of polings per time unit, i.e., the poling frequency. The average propulsive force can be calculated as.

$$F_{\text{prop}} = I_{\text{prop}} \times f. \tag{20}$$

where f is the poling frequency in Hz, i.e., the reciprocal of poling cycle time T. Equation (20) is useful to understand the trade offs between frequency and impulse that are further illustrated in Fig. 6 that presents relations between impulse, frequency, and thrust duration.

As mentioned above I_{prop} depends on the temporal excursion of both $F(t)$ and $\alpha(t)$. To separate the angle dependency from the force dependency in the expression the ratio

$$r(F(\bar{t}), \alpha(\bar{t})) = \frac{\bar{I}_{\text{prop}}}{\bar{I}_{\text{ax}}} = \frac{\int_0^1 F(\bar{t}) \times \cos\alpha(\bar{t})d\bar{t}}{\int_0^1 F(\bar{t})d\bar{t}}. \tag{21}$$

can be expressed. Then, the propulsive force, time averaged over one poling cycle, can be expressed

$$F_{\text{prop}} = r(F(\bar{t}), \alpha(\bar{t})) \times \bar{I}_{\text{ax}}(F(\bar{t})) \times f. \tag{22}$$

The corresponding mechanical power output can then be expressed as the product of the cycle-averaged propulsive force and the skier's forward velocity,

$$P_{\text{prop}} = F_{\text{prop}}\, v = r(F(\bar{t}), \alpha(\bar{t})) \times \bar{I}_{\text{ax}}(F(\bar{t})) \times f \times v. \tag{23}$$

Equations (22) and (23) form the basis for analyzing trade-offs in skiing propulsion. According to (23), the propulsive power can be increased by enhancing either the pole-force-ratio $r(F(\bar{t}), \alpha(\bar{t}))$, or the axial impulse per cycle, $\bar{I}_{\text{ax}}(F(\bar{t}))$, as well as by increasing the poling frequency f or the forward velocity v. However, these terms are not independent. As skiing speed increases, the available thrust duration within each cycle typically decreases, which tends to reduce the achievable impulse. This reduction is commonly compensated for by an increase in frequency. However, higher frequencies impose additional technical demands on the skier, often leading to further reductions in both the axial impulse and the fraction of force aligned with the direction of motion, thereby limiting the net gain in propulsive power.

3.2 Practical Ski power measurements

According to Section 3.1, skiing mechanical power may be measured by a force sensor capturing axial pole force and a motion sensor capturing the pole angle. In the double-poling technique, this implementation would yield the true mechanical power generated in propulsion. However, as illustrated in Section 2, true mechanical power does not reflect metabolic load, as gross efficiency varies with skiing conditions, primarily incline and speed. For this reason, we define ski power as

$$P_{\text{ski}} = r_0\, F_{\text{ax}} \cdot u, \tag{24}$$

where $r_0 \equiv \text{PFR}_0$ denotes the saturated value of the pole-force ratio $r(u, \gamma)$, defined as the ratio between the propulsive force and the axial pole force. Previous studies have reported the existence of such a saturation at steeper inclines [13]. Note that the term *pole-efficiency index*

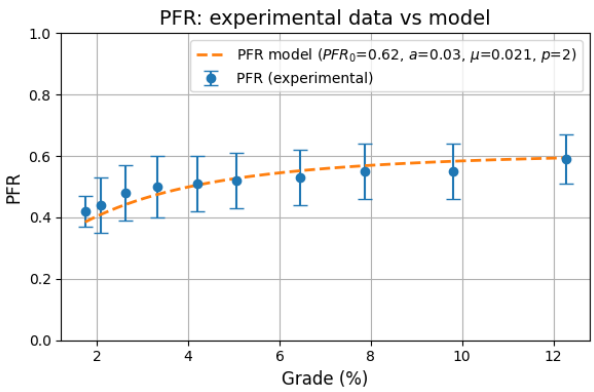


Figure 7: Pole-force-ratio (PFR) and its progression with grade

(PEI) is used in [13] for this quantity. In the present study, we adopt the term PFR, as it more transparently reflects that the quantity is not an efficiency in a strict mechanical sense, although, as shown in Section 3.3, the PFR exhibits a strong correlation with the gross efficiency (GE).

Mechanical power is proportional to metabolic power through gross efficiency (GE). Inserting P_{mech} expressed in terms of the pole-force ratio and axial pole force yields

$$\begin{aligned}\dot{V}O_2 &= \frac{P_{\text{mech}} \cdot 60}{\eta_{\text{gross}} \cdot e_{O_2}} \\ &= \frac{r(u,\gamma)\, F_{\text{ax}} \cdot 60 \cdot u}{\eta_{\text{gross}}(u,\gamma)\, e_{O_2}} \\ &= \frac{r_0\, F_{\text{ax}} \cdot u \cdot 60}{e_{O_2}} \cdot \frac{f_r(u,\gamma)}{f_\eta(u,\gamma)} \\ &= P_{\text{ski}} \cdot \frac{60}{\eta_0\, e_{O_2}} \cdot \frac{f_r(u,\gamma)}{f_\eta(u,\gamma)}.\end{aligned} \tag{25}$$

where f_r and f_η are introduced as functions describing grade and frequency dependencies of PFR and GE. respectively. Equation (25) shows that P_{ski} provides a valid proxy for metabolic load under the condition that variations in PFR and GE are aligned.

3.3 Grade and speed variations of pole-force ratio

The grade dependence of gross efficiency was examined in Section 2.2, where it was shown that GE increases with incline up to a characteristic angle, referred to as the efficiency saturation angle θ_I. Likewise, empirical studies have demonstrated that the pole-force ratio increases with increasing grade. In an early study by Holmberg and Stöggle, the ratio was reported to increase from approximately 56° at a 1° incline to about 65° at a 7° incline [13]. More recent investigations have confirmed these trends across a broader range of inclines [14].

In [14], the development of pole forces with increasing incline was investigated under conditions of constant external power output near the aerobic threshold. Figure 7 illustrates the progression of the pole force ratio (PFR) across grades. Notably, the observed trend closely resembles that described above for the gross efficiency (GE).

The dashed line represents a proposed model, given by

$$\text{PFR}(\gamma) = \frac{\text{PFR}_0}{1 + \left(\frac{a}{\gamma+\mu}\right)^p}, \tag{26}$$

where γ denotes the grade expressed as a fraction. The parameter values used are $\text{PFR}_0 = 0.62$, $a = 0.03$, $\mu = 0.021$, and $p = 2$.

The primary importance here is not the exact shape of the PFR curve, but rather the fact that it varies with grade in a manner similar to that of the gross efficiency (GE); cf. Eq. (12), which is valid under the assumption of a constant metabolic work rate at $VO_{2,\max}$ for inclines up to the critical angle θ_I.

To further highlight the similarity between gross efficiency (GE) and the pole force ratio (PFR), the dependence on grade is separated from the saturation values in accordance with Eq. (25). Figure 8 presents the resulting peak-normalized grade dependencies.

To more clearly illustrate differences in the grade dependence of f_r and f_η, the same functions are also plotted against each other, with the deviation from the identity line ($f_\eta = f_r$) shown on the right-hand axis.

4 Discussion

The aim of this work was to present a unified framework for describing energy expenditure and efficiency in cross-country skiing and roller skiing, and to demonstrate how this framework can be applied in practice for intensity control during training and, potentially, in competitive settings.

A central distinction from cycling is emphasized. In cycling, the interpretation of mechanical power is greatly simplified by the relatively constant gross efficiency (GE) across a wide range of conditions. In skiing, by contrast, GE exhibits a pronounced dependence on both speed and incline, which complicates the direct use of mechanical power as a proxy for metabolic load. In the present work, we show that these challenges can largely be addressed by characterizing how variations in GE depend on grade and velocity.

Specifically, we demonstrate that the grade dependence of GE closely mirrors that of the pole-force ratio (PFR). This alignment implies that, under practical conditions, the metabolic load can be estimated from measurements of the axial pole force and its variation with speed and incline alone. As a result, the need for explicit modeling of GE is substantially reduced in applications focused on intensity control.

For practical implementation, ski power is defined in (24), and its relationship to metabolic load is established in (25). Provided that variations in GE and PFR remain aligned, P_{ski} serves as a robust proxy for metabolic demand and can therefore be used as a basis for real-time intensity regulation in both training and applied performance monitoring.

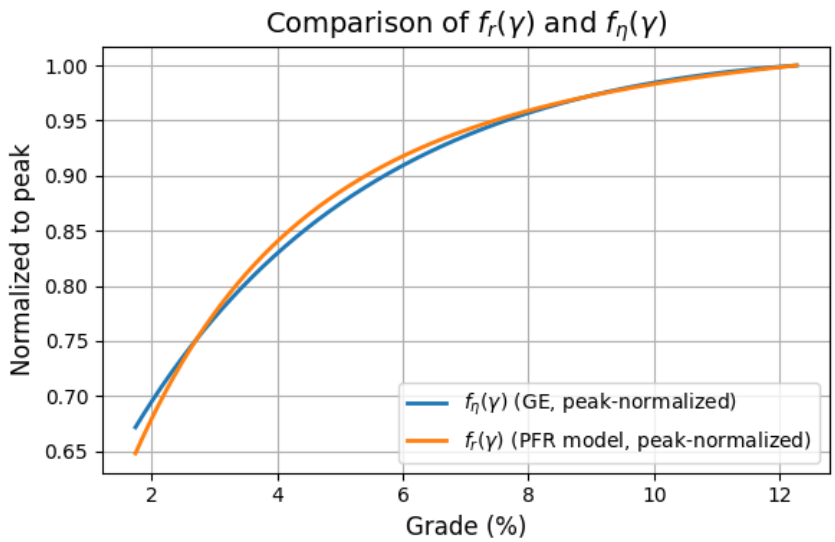
One of the primary reasons why power measurement has gained such widespread adoption in cycling is not only its utility for intensity control, but also its role as a measure of physiological capacity. Having introduced P_{ski} as a proxy for intensity regulation in skiing, it is important to recognize that it cannot be used directly as a capacity metric in the same manner as mechanical power is commonly applied in cycling.

Likewise, external mechanical power in skiing cannot be interpreted as a direct measure of capacity for the same reasons: gross efficiency varies systematically with external conditions, including incline, speed, and surface characteristics. The framework presented in this work provides a practical approach for addressing these challenges.

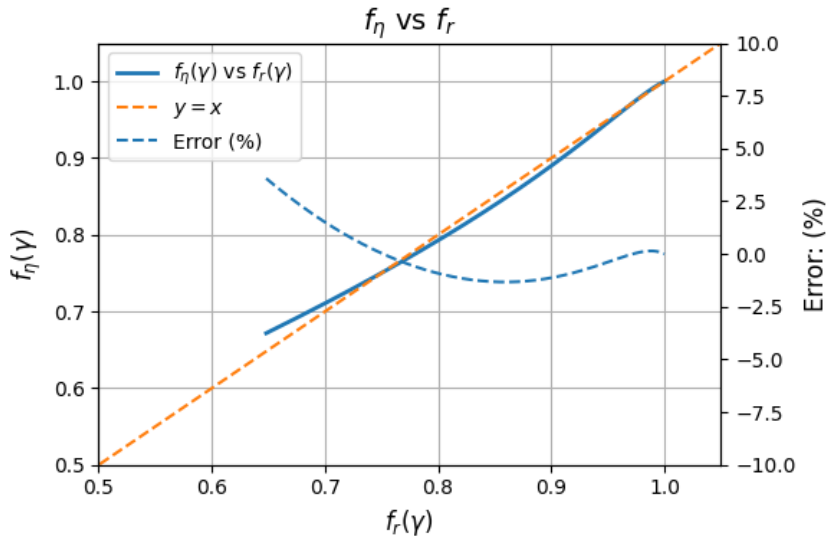
In principle, only one additional element is required to translate P_{ski} into a capacity-related metric. Each skier must characterize their individual pole-force ratio (PFR) and its dependence on grade, speed, and frictional conditions. Once this relationship is known, the corresponding external mechanical power can be estimated. This external power then serves as a direct measure of capacity, albeit one that is specific to a given combination of speed and incline.

The advantage of P_{ski} is that it inherently accounts for variations in grade and speed, thereby providing a more global measure of performance. For greater precision, the PFR should be normalized at the individual level, as both its absolute values and its variation are, to some extent, technique-dependent.

A limitation of the present framework is the assumption of a stable relationship between PFR, technique, and external conditions. In practice, variations in snow properties, pole–snow



(a) Peak-normalized grade dependence of the resistive factor $f_r(\gamma)$ (derived from the PFR model) and the efficiency factor $f_\eta(\gamma)$ (derived from the GE model).



(b) Direct comparison of $f_\eta(\gamma)$ versus $f_r(\gamma)$. The dashed line indicates the identity relation $f_\eta = f_r$, while the right-hand axis shows the deviation $f_\eta - f_r$.

Figure 8: Comparison of the peak-normalized grade dependence of gross efficiency and pole force ratio. (a) Normalized functions $f_r(\gamma)$ and $f_\eta(\gamma)$ plotted as a function of grade. (b) Parametric representation of f_η versus f_r, highlighting deviations from the identity relation and thereby differences in their respective grade dependencies.

interaction, and fatigue-induced changes in technique may introduce additional variability that is not explicitly captured by the model. Future work should therefore focus on assessing the robustness of the proposed relationships under field conditions and across a broader range of skiing techniques and athlete populations, as well as on establishing robust individual normalization procedures.

One aspect not addressed in the present work is the extension of the framework to skiing techniques that involve both upper and lower body propulsion, such as diagonal stride, kick-double poling, and various freestyle gears. Although the drivetrain of these techniques is more complex, since not all mechanical work is transmitted through the poles, preliminary studies indicate that the axial pole force scales with overall work rate. This observation suggests that a pole-force ratio can also be defined for these techniques and used as a basis for power estimation and intensity control.

In a future implementation, motion patterns governing transitions between gears may be described in terms of parameters such as cycle rate, impulse, and thrust duration, in combination with geo-referenced data providing speed and incline. Together, these elements point toward a unified framework for power-based intensity control and capacity screening in cross-country skiing.

An added contribution of this work is the explicit estimation of metabolic load as a function of speed and grade, enabled by the first-principles $\dot{V}O_2$ model derived in Eq. (16) and summarized in Table 1. By factorizing the dependence on velocity and incline, the framework provides a transparent and physically grounded mapping between external conditions and metabolic demand.

In analogy with the metabolic equations for treadmill running reported in the American College of Sports Medicine guidelines [15] and physically motivated in [3], this representation facilitates both practical intensity prescription and the interpretation of performance across heterogeneous terrain.

5 Conclusion

This work develops a unified analytical framework for describing external mechanical power, metabolic energy expenditure, and gross efficiency in cross-country skiing across a wide range of terrain and technique-dependent conditions. By explicitly accounting for the non-zero resistive power demands associated with ski–snow friction and aerodynamic drag, the model highlights a fundamental distinction between skiing and level terrestrial locomotion, and provides a physically interpretable basis for quantifying performance in both level and inclined motion.

The introduction of an incremental incline efficiency and a skiing economy parameter enables a separation between external resistive work and internal dissipative processes, yielding closed-form expressions for oxygen uptake and gross efficiency that remain analytically tractable while capturing the experimentally observed saturation behavior at moderate to steep grades. Validation against published treadmill and roller-ski studies demonstrates that the framework reproduces key empirical trends in efficiency and metabolic demand without requiring technique-specific calibration.

Beyond its theoretical implications, the model offers practical utility through simplified tabular representations that allow rapid estimation of speed, power, and metabolic cost under typical training and competition conditions. These results provide a quantitative foundation for performance analysis, equipment evaluation, and the interpretation of field-based power and physiological measurements in cross-country skiing.

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Table 1: Predicted speed v (km/h) as a function of oxygen uptake VO_2 and treadmill incline θ using (17). Parameters: RMR = 3.5 $\mathrm{mL\,kg^{-1}\,min^{-1}}$, SE = 30 $\mathrm{mL\,kg^{-1}\,km^{-1}}$, $\mu = 0.02$, $\gamma_I = 0.15$, $\eta_{I0} = 0.22$, $p = 3$, $g = 9.81\ \mathrm{m\,s^{-2}}$, $e_{O_2} = 20.9\ \mathrm{J\,mL^{-1}}$.

VO_2 ($\mathrm{mL\,kg^{-1}\,min^{-1}}$)	Treadmill incline θ (deg)										
	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
30	20.7	14.0	10.5	8.4	7.0	6.0	5.2	4.6	4.1	3.7	3.3
32	22.3	15.0	11.3	9.1	7.5	6.5	5.6	5.0	4.4	4.0	3.6
34	23.9	16.1	12.1	9.7	8.1	6.9	6.0	5.3	4.7	4.2	3.8
36	25.4	17.1	12.9	10.3	8.6	7.4	6.4	5.7	5.0	4.5	4.1
38	27.0	18.2	13.7	11.0	9.1	7.8	6.8	6.0	5.3	4.8	4.3
40	28.6	19.2	14.5	11.6	9.7	8.3	7.2	6.4	5.7	5.1	4.6
42	30.1	20.3	15.3	12.2	10.2	8.7	7.6	6.7	6.0	5.4	4.8
44	31.7	21.3	16.1	12.9	10.7	9.2	8.0	7.1	6.3	5.6	5.1
46	33.3	22.4	16.9	13.5	11.3	9.6	8.4	7.4	6.6	5.9	5.3
48	34.8	23.4	17.7	14.1	11.8	10.1	8.8	7.7	6.9	6.2	5.6
50	36.4	24.5	18.4	14.8	12.3	10.5	9.2	8.1	7.2	6.5	5.8
52	38.0	25.6	19.3	15.4	12.8	11.0	9.6	8.4	7.5	6.7	6.1
54	39.5	26.6	20.0	16.1	13.4	11.4	10.0	8.8	7.8	7.0	6.3
56	41.1	27.7	20.8	16.7	13.9	11.9	10.4	9.1	8.1	7.3	6.6
58	42.7	28.7	21.6	17.3	14.4	12.3	10.7	9.5	8.4	7.6	6.8
60	44.2	29.8	22.4	18.0	15.0	12.8	11.1	9.8	8.8	7.9	7.1
62	45.8	30.8	23.2	18.6	15.5	13.2	11.5	10.2	9.1	8.1	7.3
64	47.3	31.9	24.0	19.2	16.0	13.7	11.9	10.5	9.4	8.4	7.6
66	48.9	32.9	24.8	19.9	16.6	14.1	12.3	10.9	9.7	8.7	7.8
68	50.5	34.0	25.6	20.5	17.1	14.6	12.7	11.2	10.0	9.0	8.1
70	52.0	35.0	26.4	21.1	17.6	15.1	13.1	11.6	10.3	9.2	8.3
72	53.6	36.1	27.2	21.8	18.1	15.5	13.5	11.9	10.6	9.5	8.6
74	55.2	37.2	28.0	22.4	18.7	16.0	13.9	12.3	10.9	9.8	8.8
76	56.7	38.2	28.8	23.0	19.2	16.4	14.3	12.6	11.2	10.1	9.1
78	58.3	39.3	29.6	23.7	19.7	16.9	14.7	13.0	11.6	10.3	9.3
80	59.9	40.3	30.4	24.3	20.3	17.3	15.1	13.3	11.9	10.6	9.6
82	61.4	41.4	31.2	24.9	20.8	17.8	15.5	13.7	12.2	10.9	9.8
84	63.0	42.4	32.0	25.6	21.3	18.2	15.9	14.0	12.5	11.2	10.1
86	64.6	43.5	32.8	26.2	21.9	18.7	16.2	14.4	12.8	11.5	10.3
88	66.1	44.5	33.6	26.9	22.4	19.1	16.6	14.7	13.1	11.7	10.6
90	67.7	45.6	34.4	27.5	22.9	19.6	17.0	15.1	13.4	12.0	10.8
92	69.3	46.6	35.1	28.1	23.4	20.0	17.4	15.4	13.7	12.3	11.1
94	70.8	47.7	35.9	28.8	24.0	20.5	17.8	15.7	14.0	12.6	11.4
96	72.4	48.7	36.7	29.4	24.5	20.9	18.2	16.1	14.3	12.9	11.6
98	74.0	49.8	37.5	30.0	25.0	21.4	18.6	16.4	14.7	13.1	11.9
100	75.5	50.8	38.3	30.7	25.6	21.8	19.0	16.8	15.0	13.4	12.1