A Hybrid First-Principles Model for Understanding Running Energetics on Flat and Inclined Terrain

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Abstract

Running power has emerged as a potential tool for exercise monitoring in running, inspired by its established role in cycling. Unlike cycling, where external mechanical power is directly measured, running power is inferred from kinematic proxies, leading to conceptual ambiguity and substantial inconsistency between devices. This ambiguity arises not only from measurement limitations, but also from fundamental differences in how mechanical work and efficiency manifest in running. In cycling, gross efficiency remains approximately constant across flat and inclined terrain due to a well-defined external power output. In running, gross efficiency is, by definition, effectively zero on level ground, where metabolic cost is dominated by internal energy dissipation rather than by external mechanical work defined as displacement against a restraining force. Only during uphill running does a substantial and well-defined mechanical work component emerge, allowing power-based interpretations to become mechanically meaningful.

This work reviews running energetics and established empirical models, including the American College of Sports Medicine (ACSM) metabolic model for treadmill running and performance-based approaches such as the Cooper test. These models are placed in the context of early foundational work by Margaria, as well as more recent practical validation of running power by Taboga et al.

The main contribution of this work is the formulation of a more intuitive and physically interpretable model of running energetics, explicitly grounded in measurable quantities such as running economy and efficiency. Particular attention is given to efficiency, identified as the central challenge in discussions of running energetics due to its strong variability and the ambiguous definition of external work rate in level running. Based on mechanical reasoning, internal limb losses are lumped into an equivalent restraining force acting in the direction of motion, enabling the definition of a quasi-gross efficiency that links the ambiguous concept of running power to well-established metabolic and performance-based models.

Building on this framework, a hybrid first-principles model (HFPM) is introduced for the description of running energetics on level and graded terrain. The model combines a physics-based representation of mechanical work associated with incline with an empirically grounded performance-VO_2 relationship for level running. The resulting formulation retains the linear structure of the ACSM treadmill equation while providing a mechanistic interpretation of how its known limitations can be understood as systematic variations in efficiency and running economy across speed and slope.

Finally, the proposed framework is summarized in user-friendly tabulated data relating running performance, expressed as equivalent VO_2, across ranges of speed and grade. These tables are intended to facilitate practical application, enabling direct interpretation for indoor treadmill conditions without air resistance as well as for outdoor running through the inclusion of simple aerodynamic corrections.

1 Introduction

Relationships between mechanical work and metabolic energy expenditure are central to exercise physiology and have been investigated for over a century, with early quantitative formulations dating back to the work of A. V. Hill and contemporaries [1]. Fundamentally, these relationships describe how mechanical work performed by the musculoskeletal system is supported by metabolic energy turnover, typically quantified through oxygen uptake.

The complexity of this relationship depends strongly on the nature of the external work performed. In cycling, for example, the mechanical work rate is well defined and directly measurable at the drivetrain. As a result, the relationship between mechanical power output and metabolic energy expenditure can be described in terms of gross efficiency, which remains relatively stable across a wide range of workloads and gradients [2, 3]. This has enabled power to become a robust and widely adopted metric for both performance analysis and intensity control[4].

In running, the situation is fundamentally more complex. Unlike cycling, the external mechanical work performed during running, particularly on level terrain, is not straightforward to define. A substantial portion of the metabolic cost is associated with internal work, elastic energy storage, and dissipation, rather than net external mechanical work. As a consequence, work–energetics studies in running have traditionally relied on empirical descriptions of metabolic cost rather than explicit mechanical formulations.

One of the most practically applied empirical approaches is the Cooper test, a 12-minute running test that establishes a relationship between covered distance and maximal oxygen uptake [5]. While the Cooper test is limited to level running, more comprehensive investigations have examined how metabolic cost depends on both running speed and incline. Pioneering work by Margaria and colleagues established approximately linear relationships between oxygen uptake, speed, and slope during treadmill running [6]. These findings form the empirical foundation of the American College of Sports Medicine (ACSM) treadmill equation, which remains one of the most widely used models for estimating oxygen uptake as a function of running speed and incline in laboratory and clinical settings [7].

Despite its widespread use, the ACSM treadmill equation has been shown to exhibit systematic limitations, particularly at high running speeds and steep inclines [8, 9]. From a physical point of view these variations are expected as the linear assumptions cannot be expected to be valid over a wide range of variations in speed and incline.

The purpose of the present work is to establish an intuitive and mechanically interpretable relationship between metabolic energy expenditure, running speed, and incline, with explicit emphasis on the roles of mechanical work, efficiency, and running economy. Building on Margaria's empirical description of the metabolic cost of level running, the additional energetic demand associated with incline is introduced through a first-principles mechanical formulation of gravitational work. This contribution is translated into metabolic cost using an explicitly defined incline-referred incremental efficiency, which is further reformulated into an equivalent gross efficiency that approaches a plateau at large inclines in line with Minetti and Winters [10, 11]. Together, the above formulated hybrid framework preserves the accuracy and simplicity of existing empirical models while providing improved conceptual transparency regarding how variations in efficiency and running economy underlie their applicability and limitations. To facilitate practical application, the results of the proposed framework are further summarized in user-oriented tabulated data, relating running performance expressed as equivalent VO_2 across ranges of speed and incline. These tables are intended to bridge the gap between theoretical formulation and applied use, enabling direct interpretation in both laboratory treadmill settings and field-based conditions.

Within the same energetic framework, additional external work contributions relevant in outdoor running, such as aerodynamic drag, can be incorporated, extending the applicability of the model beyond controlled environments. This representation is shown to be consistent with classical field-based performance models, including the Cooper test.

This work is organized as follows. Section 2 establishes the theoretical framework for power and energy considerations in running, including work-energy metrics, unit conventions, and the definitions of gross, net, incremental, and quasi-gross efficiency that are needed to interpret running power in a mechanically meaningful way. Section 3 introduces the hybrid first-principles model (HFPM) for running energetics on level and inclined terrain by combining an empirically grounded description of level-running cost (expressed via running economy) with a first-principles formulation of gravitational work on incline; the extension to outdoor running through aerodynamic drag is also presented. Section 4 discusses practical applications of the framework for training analysis and performance monitoring and provides user-oriented tables that map between running speed, incline/grade (or vertical ascent rate), equivalent VO_2, and the corresponding power-based interpretations under both treadmill and outdoor conditions. Finally, Section 5 summarizes the main conclusions and outlines perspectives for future work.

2 Theoretical framework for power and energy considerations

The aim of this section is to establish a theoretical framework linking running mechanics to metabolic energy expenditure.

2.1 Work-energy metrics, units, and conversions

A persistent challenge in review of the exercise physiology literature is that units and conventions used to describe energetic cost and work rate often deviate from standard SI units. Although these quantities represent the same underlying physical concepts, differences in notation, normalization, and reference frames can hinder direct comparison between studies.

One of the central metrics in exercise physiology is work rate, defined as mechanical work performed per unit time. From a mechanical perspective, work rate corresponds to power and is expressed in SI units of watts (W),

$$1\ \mathrm{W} = 1\ \mathrm{J\,s^{-1}} = 1\ \mathrm{N\,m\,s^{-1}}. \tag{1}$$

Starting from first principles, mechanical work is defined as the line integral of force along a displacement. For the case of a constant force acting along the direction of motion, this reduces to

$$W = F\,d, \tag{2}$$

and power is obtained as the time derivative of work,

$$P = \frac{dW}{dt} = F\,u, \tag{3}$$

where u (m/s) denotes the velocity of the center of mass.

In exercise physiology, mechanical work rate is frequently reported as energy per unit body mass and time, for example in $\mathrm{J\,kg^{-1}\,min^{-1}}$ or $\mathrm{kcal\,kg^{-1}\,h^{-1}}$, rather than in the SI unit of power, W. Metabolic rate is commonly inferred from oxygen uptake and converted to energy using the energetic equivalent of oxygen, $e_{O_2} \approx 20.90\,\mathrm{kJ\,L^{-1}}$, according to

$$P_{\mathrm{met}} = \frac{\dot{V}_{O_2}\,e_{O_2}}{60}, \tag{4}$$

where $\dot{V}_{O_2}$ is expressed in $\mathrm{mL\,kg^{-1}\,min^{-1}}$, yielding P_{met} in $\mathrm{W\,kg^{-1}}$. This formulation presents mechanical and metabolic descriptions in consistent dimensions, facilitating subsequent comparison and the definition of efficiency.

Equations (2-3) present the first-principles mechanics governing the fundamental quantities force, work, and power. By definition, the unit of energy is independent of the physical context in which it is applied; however, as noted above, a variety of alternative units and normalizations are commonly used in the exercise physiology literature. For example, total energetic cost is frequently reported per unit distance traveled, typically in $\mathrm{J\,kg^{-1}\,m^{-1}}$, where distance refers to displacement along the direction of motion.

Table 1: Summary of mechanical and energetic quantities, their SI units, commonly used alternative units, and typical normalizations in exercise physiology.

Quantity	SI unit	Alternative units	Normalizations
Force F	N	$\mathrm{kg\,m\,s^{-2}}$	$\mathrm{N\,kg^{-1}}$ ($\mathrm{m\,s^{-2}}$)
Work W	J (N m)	1 kcal = 4184 J	$\mathrm{J\,kg^{-1}}$
Power P	W	$1\ \mathrm{kcal\,min^{-1}} = \frac{4180}{60}\ \mathrm{W}$	$\mathrm{W\,kg^{-1}}$
Metabolic power P_{met}	W	$1\ \mathrm{L\,min^{-1}} = \frac{20.9\times10^3}{60}\ \mathrm{W} \approx 350\ \mathrm{W}$	$\mathrm{W\,kg^{-1}}$
Running economy (RE)	$\mathrm{J\,m^{-1}}$	$1\ \mathrm{mL\,km^{-1}} = \frac{1000}{20.9}\ \mathrm{J\,m^{-1}}$	$\mathrm{J\,kg^{-1}\,m^{-1}}$, $\mathrm{mL\,kg^{-1}\,km^{-1}}$

Expressed in this form, the mass-normalized energetic cost per unit distance is dimensionally equivalent to $\mathrm{N\,kg^{-1}}$, and thus to an acceleration ($\mathrm{m\,s^{-2}}$). This equivalence follows directly from the definition of work as force acting over a displacement. However, despite this dimensional correspondence, the energetic cost per unit distance does not, in general, represent a net mechanical force or acceleration. For example, during steady-state level running the net force and net acceleration along the direction of motion vanish, even though the energetic cost per unit distance remains non-zero. In this case, no unique external mechanical power can be defined, as the metabolic expenditure is instead associated with cyclic breaking-propulsion dynamics and internal dissipative processes.

In studies of inclined locomotion, a well-defined mechanical work is performed against gravity, associated with vertical displacement. This gravitational work is given by

$$W_{\mathrm{grav}} = mgh, \tag{5}$$

where h denotes the vertical displacement of the center of mass. The mass-normalized energetic cost associated with this work can be expressed per unit vertical distance in $\mathrm{J/kg/m_v}$, where the subscript v denotes vertical displacement.

Energy expenditure may further be normalized by body mass and running speed to define the cost of transport as

$$C = \frac{P}{m\,u}, \tag{6}$$

which, as a matter of fact, is likewise expressed in $\mathrm{J\,kg^{-1}\,m^{-1}}$. In applied sport science, C is commonly converted to an oxygen equivalent per unit distance and body mass, often referred to as running economy (RE), which can be calculated as

$$\mathrm{RE} = 1000\frac{C}{e_{\mathrm{O_2}}}, \tag{7}$$

where RE is expressed in $\mathrm{mL\,kg^{-1}\,km^{-1}}$.

Table 1 summarizes some commonly used work-energy and metabolic metrics in running physiology together with their primary unit conversions.

2.2 Mechanical efficiency in human locomotion

Mechanical efficiency describes the relationship between external mechanical work and the metabolic energy required to perform that work. Multiple definitions of efficiency are used in the literature, and careful distinction between them is essential when analysing locomotion energetics. A comprehensive review of efficiency definitions and their physiological interpretation is provided by Cavagna [12]. For convenience, this section provides an overview of the definitions most relevant to the present work.

Gross efficiency (GE) represents the most fundamental definition of efficiency and is defined as the ratio between total external mechanical work and metabolic energy expenditure. When expressed in terms of power, gross efficiency can be written as

$$\eta_{\text{gross}} = \frac{P_{\text{mech}}}{P_{\text{met}}}. \tag{8}$$

Net efficiency (NE) is defined as the ratio between external mechanical power and the *incremental* metabolic power required to perform the mechanical task, obtained by subtracting a baseline metabolic rate from the total metabolic power. Expressed in terms of power, net efficiency is given by

$$\eta_{\text{net}} = \frac{P_{\text{mech}}}{P_{\text{met}} - P_{\text{baseline}}}, \tag{9}$$

where P_{baseline} represents the metabolic power not directly attributable to mechanical work production, often referred to rest-metabolic-rate (RMR) that will be discussed later.

Incremental efficiency (NE) describes the additional metabolic cost required to produce an incremental amount of mechanical power on top of an already non-zero work rate. It is defined as

$$\eta_{\text{inc}} = \frac{\Delta P_{\text{mech}}}{\Delta P_{\text{met}}}, \tag{10}$$

where the increments ΔP_{mech} and ΔP_{met} are evaluated between two steady-state conditions.

Incremental efficiency may be defined with respect to changes in speed or incline, while holding the other variable constant. In uphill locomotion, variations in incline at constant speed provide a particularly clear mechanical interpretation, as changes in metabolic power are directly associated with changes in vertical mechanical work. Then, the incline-referred incremental efficiency (η_I) can be defined as

$$\eta_I = \left. \frac{\partial p_{\text{mech,grav}}}{\partial p_{\text{met,grav}}} \right|_{v=\text{const}}, \tag{11}$$

where $p_{\text{mech,grav}} = gu \sin\theta$ is the mechanical power per unit mass associated with vertical lifting of the center of mass, and $p_{\text{met,grav}} = e_{O_2}\, VO_{2,\text{grav}}$ is the corresponding metabolic power per unit mass.

Need for different efficiency definitions. For well-trained athletes, the difference between gross and net efficiency is typically small ($\sim$5%), as resting metabolic power is small compared to the metabolic power required to perform external mechanical work. As a result, net efficiency is seldom used in applied exercise testing, since gross efficiency is more straightforward to define and does not require an independent assessment of resting metabolism.

Incremental efficiency, on the other hand, is commonly used to quantify the additional metabolic cost associated with an incremental increase in mechanical work on top of an established work rate. This approach is particularly useful in situations where gross efficiency varies with external load. A prominent example is running, where gross efficiency is effectively zero on level terrain due to vanishing net external work, but increases with incline as mechanical work against gravity becomes proportional to vertical displacement. In contrast, gross efficiency in cycling is approximately constant, typically around 20-25%, owing to the well-defined external mechanical work at the crank.

Incline-saturated gross efficiency and running power: As discussed above, gross efficiency (GE) is fundamentally ill-defined during level running, as the net external mechanical work vanishes according to (8). In the absence of a uniquely defined mechanical work term, mechanical power likewise becomes ambiguous, rendering the concept of *running power* inherently vague. Over the past decade, this ambiguity has led to a proliferation of incompatible definitions, as different commercial and academic approaches have introduced device-specific running power metrics based on kinematic proxy variables rather than on first-principles mechanical formulations [13, 14].

To establish a consistent efficiency-based reference, we build on the well-established observation that GE during uphill running increases with grade and approaches a saturation value at sufficiently steep inclines. Multiple experimental studies have demonstrated that this *incline-saturated gross efficiency* becomes approximately constant for grades exceeding about 30% and assumes values in the range $\eta_{\mathrm{G,sat}} \approx$ 20-25 % [6, 10]. In this regime, metabolic energy expenditure is dominated by well-defined mechanical work performed against gravity, rendering efficiency directly interpretable in the classical mechanical sense. Experimental studies of uphill locomotion, notably those by Margaria and later by Minetti, further show that the incremental energetic cost per unit vertical displacement remains approximately constant over a wide range of positive slopes, supporting the use of a mechanically defined vertical work term combined with an empirically determined efficiency.

We interpret this incline-saturated efficiency as representing the intrinsic efficiency of positive muscular work and adopt it here as a reference conversion factor between metabolic power and an equivalent mechanical running power. While this efficiency is mechanically well defined primarily under steep uphill conditions, we assume that it also provides a physically meaningful conversion factor during level running, despite the fact that external mechanical work is not uniquely defined in that case.

Running power may then be defined as

$$P_{\mathrm{run}} = \eta_{\mathrm{G}}^{*} \, \frac{20.9 \, \dot{V}O_2}{60}, \tag{12}$$

where $\dot{V}O_2$ is expressed in $\mathrm{mL\,kg^{-1}\,min^{-1}}$ and η_{G}^{*} denotes a quasi-gross efficiency, assumed to be constant across terrain conditions. Throughout this work, a value of $\eta_{\mathrm{G}}^{*} = 25\,\%$ is adopted, consistent with empirical observations of the saturated gross efficiency $\eta_{\mathrm{G,sat}}$.

It should be noted that $\eta_{\mathrm{G}}^{*} = \eta_{\mathrm{G,sat}}$ does not constitute a gross efficiency in the strict mechanical sense. Rather, it should be regarded as an effective conversion factor that enables a physically interpretable and internally consistent reference for running power. This interpretation is subsequently reconciled with explicit mechanical work calculations in Section 3 through the hybrid first-principles treadmill model.

2.3 Empirical models of running energetics

Work-energy relations in human locomotion have been studied for more than a century [1], specifically in running [5, 6, 15]

Work-energy relations in treadmill running. Among the first to systematically quantify the energetic cost of running as a function of both speed and incline was Margaria [6]. His pioneering work demonstrated that, for a given treadmill incline, energy expenditure increases approximately linearly with running speed, while the slope of this relationship depends strongly on grade.

Figure 1 shows oxygen uptake as a function of running speed for different treadmill grades, extracted from Margaria's original data but converted from $\mathrm{kcal\,kg^{-1}\,h^{-1}}$ to $\mathrm{mL\,kg^{-1}\,min^{-1}}$ using the unit conversions described in Section 2.1. For each incline, the relationship between speed

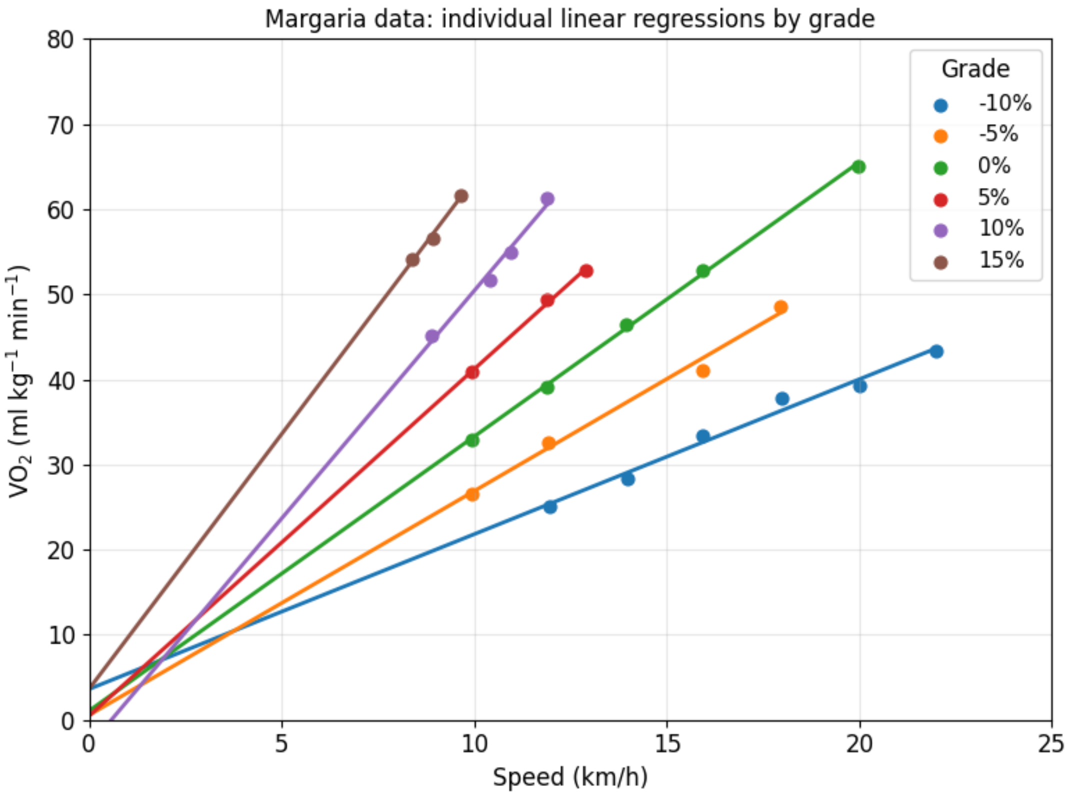


Figure 1: Oxygen uptake (VO_2) as a function of running speed at different treadmill grades, based on data reported by Margaria but converted from kcal/kg/min to more contemporary ml/kg/min according to conversion relations in Section 2.1. For each incline, a linear relationship between VO_2 and speed is observed, with steeper slopes at higher grades.

and oxygen uptake is well described by a linear model of the form

$$\dot{V}_{O_2} = k\,v + m, \tag{13}$$

where v denotes running speed in $\mathrm{km\,h^{-1}}$ and $\dot{V}_{O_2}$ is expressed in $\mathrm{mL\,kg^{-1}\,min^{-1}}$.

As shown in Fig. 1, the fitted slope parameter k increases systematically with grade, reflecting the growing energetic contribution of vertical work against gravity. The best-fit coefficients for each incline are summarized in Fig. 2, which reveals an approximately linear dependence of k on grade. Taken together, these observations suggest that oxygen uptake may be expressed as a separable function of speed and incline according to

$$\mathrm{VO_2}(v, \mathrm{grade}) \approx \big(m_g + k_g\,\mathrm{grade}\big)\,v + m_0, \tag{14}$$

where m_g and k_g are empirical coefficients obtained from regression.

Equation (14) has the same functional form as the widely used ACSM treadmill model, originally expressed as [7]

$$\mathrm{VO_2} = 0.2\,v_m + 0.9\,v_m \cdot \mathrm{grade} + 3.5, \tag{15}$$

where v_m is running speed in $\mathrm{m\,min^{-1}}$ and grade is expressed as a fraction (e.g. 0.10 for a 10% incline). Rewriting (15) using speed in $\mathrm{km\,h^{-1}}$ ($1\ \mathrm{km\,h^{-1}} = 16.67\ \mathrm{m\,min^{-1}}$) and expressing grade in percent yields

$$\mathrm{VO_2} = 3.33\,v + 0.15\,v \cdot \mathrm{grade} + 3.5, \tag{16}$$

which is notably similar to the empirical regressions derived from Margaria's data, Figure 2.

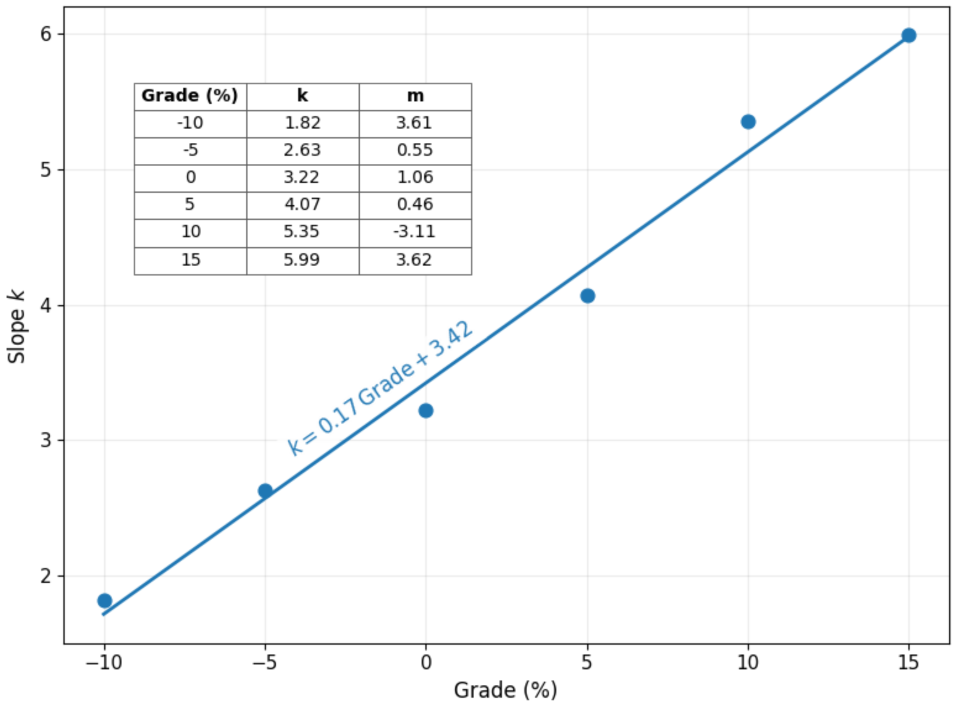


Figure 2: Linear regression parameters from Fig. 1 and illustration of linear relationship between treadmill grade and the fitting parameter k in (13).

The primary difference between the ACSM model parameters and the direct linear regressions derived from Margaria's original data lies in the constant term m_0. In the ACSM model, $m_0 = 3.5\ \mathrm{mL\,kg^{-1}\,min^{-1}}$ represents resting metabolic demand, whereas the per-grade regressions obtained from Margaria's data yield intercept values with a mean of $m_0 \approx 1.0$ and a standard deviation of $\pm 2.3\ \mathrm{mL\,kg^{-1}\,min^{-1}}$. This relatively large scatter reflects the sensitivity of the intercept to limited data precision, as the regression is constrained by measurements obtained at running speeds well above zero. From a physical perspective, there is little justification for allowing m_0 to vary with grade. Instead, m_0 should be interpreted as a fixed offset associated with resting metabolism, consistent with the structure of the ACSM treadmill equation.

With respect to the remaining parameters, the difference in m_g between the models is marginal, while the difference in k_g is less than 10% when a fixed intercept $m_0 = 3.5\ \mathrm{mL\,kg^{-1}\,min^{-1}}$ is imposed. However, before discussing these differences in detail, it should be emphasized that the two-step linear regression procedure, although pedagogically useful, is not the most accurate approach for obtaining a global fit of the model parameters. A more rigorous method is to minimize the least-squares error by fitting all data points simultaneously to Eq. (14).

Table 2 summarizes the regression coefficients obtained using four different fitting strategies. Notably, the coefficients resulting from the global fit with fixed $m_0 = 3.5\ \mathrm{mL\,kg^{-1}\,min^{-1}}$ are nearly identical to those of the ACSM equation given in (16).

To illustrate the agreement for the different regression methods, Fig. 3 presents regression lines for four different regression methods.

Level running metabolism and the Cooper test Although the ACSM treadmill equation, rooted in Margaria's pioneering work, remains one of the most widely used empirical relations for describing the energetic cost of running across different slopes, several alternative performance–VO_2 relationships exist. In the context of level running, where external mechanical work is not uniquely defined, such empirical relations provide a practical means of linking running performance to metabolic demand. One particularly practical and widely applied approach

Table 2: Regression coefficients for the model $VO_2 = (m_g + k_g\,\text{grade})\,v + m_0$, obtained using four different regression methods. The coefficient k_g is given in mL/kg/min/(km/h)/%, m_g in mL/kg/min/(km/h), and m_0 in $\mathrm{mL\,kg^{-1}\,min^{-1}}$.

Regression method	k_g	m_g	m_0
Two-step, free m_0	0.171 ± 0.00910	3.42 ± 0.0809	1.03 ± 2.49
Two-step, $m_0 = 3.5$ (fixed)	0.164 ± 0.00402	3.28 ± 0.0358	3.50 (fixed)
Global LSQ, free m_0	0.148 ± 0.00694	3.16 ± 0.202	4.54 ± 2.67
Global LSQ, $m_0 = 3.5$ (fixed)	0.150 ± 0.00469	3.23 ± 0.0377	3.50 (fixed)

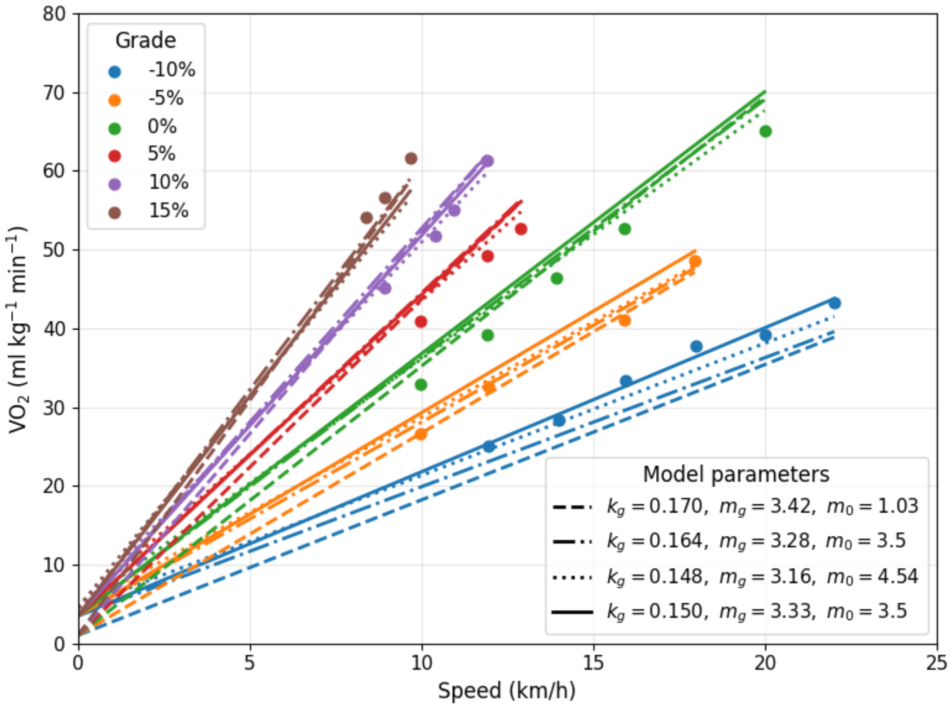


Figure 3: Illustration of agreement for different regression methods versus data in Margaria's original work [6].

is the Cooper test, which is commonly used to estimate aerobic capacity from field running performance.

The Cooper test was proposed in 1968 [5] and consists of a 12.00 min all-out run in which the total distance d (in meters) is recorded. From this single performance measure, maximal oxygen uptake ($VO_{2,\max}$) can be estimated using the empirical relation

$$VO_{2,\max}\,[\mathrm{mL\,kg^{-1}\,min^{-1}}] = \frac{d - 504.9}{44.73}. \tag{17}$$

The Cooper relation can be reformulated in terms of running speed by noting that the mean flat-ground speed during the test is given by $v_{\text{flat}} = \frac{d/1000}{12/60}\quad [\mathrm{km\,h^{-1}}]$, which may be rearranged to $d = 200\,v_{\text{flat}}$. Allowing (17) to be expressed

$$VO_{2,\max} = \frac{200}{44.73}\,v_{\text{flat}} - \frac{504.9}{44.73} \approx 4.47\,v_{\text{flat}} - 11.29, \tag{18}$$

providing a direct empirical relationship between sustainable flat-ground running speed and aerobic capacity.

Equation (18) further allows conversion between running speed and equivalent times over fixed distances, such as 3000 m [16], which is a common practical variant of the Cooper test used in applied and field-based settings.

Table 3 summarises Cooper-test performances in the 2500–4800 m range, reporting the estimated $VO_{2,max}$ from Eq. (17), the corresponding flat-ground speed, equivalent 3000 m time, and the associated mechanical work rate according to (12).

Table 3: Cooper test: distance, 3000 m time, speed, estimated VO_2max, and running power according to Eq. (12).

Distance d (m)	Time 3000 m (mm:ss)	v_{flat} ($\mathrm{km\,h^{-1}}$)	VO_2max ($\mathrm{mL\,kg^{-1}\,min^{-1}}$)	P_{run} ($\mathrm{W\,kg^{-1}}$)
2500	14:24	12.5	44.6	3.88
2600	13:51	13.0	46.8	4.08
2700	13:20	13.5	49.1	4.27
2800	12:51	14.0	51.3	4.47
2900	12:25	14.5	53.6	4.66
3000	12:00	15.0	55.8	4.86
3100	11:37	15.5	58.0	5.05
3200	11:15	16.0	60.3	5.25
3300	10:55	16.5	62.5	5.44
3400	10:35	17.0	64.7	5.64
3500	10:17	17.5	67.0	5.83
3600	10:00	18.0	69.2	6.03
3700	09:44	18.5	71.4	6.22
3800	09:28	19.0	73.7	6.42
3900	09:14	19.5	75.9	6.61
4000	09:00	20.0	78.1	6.80
4100	08:47	20.5	80.4	7.00
4200	08:34	21.0	82.6	7.19
4300	08:22	21.5	84.8	7.39
4400	08:11	22.0	87.1	7.58
4500	08:00	22.5	89.3	7.78
4600	07:50	23.0	91.6	7.97
4700	07:40	23.5	93.8	8.17
4800	07:30	24.0	96.0	8.36

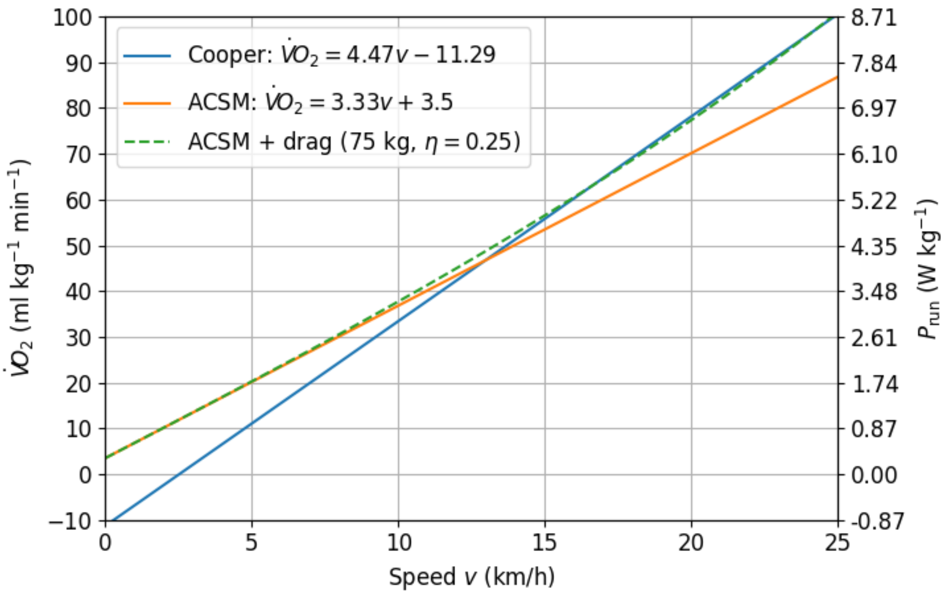


Figure 4: Comparison between linear equations describing the level-running VO_2-speed relationship, $VO_2 = k\,v + m$.

Running model comparison: Both the ACSM treadmill equation and the Cooper test describe linear relationships between oxygen uptake and running speed on level ground, each of which can be written on the form $VO_2 = k\,v + m$. However, the fitted coefficients differ substantially between the two models, cf. (16) and (18), causing their predictions to coincide only over a limited range of running speeds. This discrepancy does not reflect an inconsistency in underlying physiological principles, but rather the fact that the two models were developed for fundamentally different running scenarios.

The ACSM model was formulated for treadmill running under controlled laboratory conditions, where air resistance is negligible and the energetic cost is dominated by internal mechanical work and, in the case of incline, by mechanical work against gravity. The Cooper test, on the other hand, was designed as a field-based assessment of aerobic capacity derived from outdoor running performance, where aerodynamic drag contributes increasingly to the metabolic demand with speed. As noted above, the Cooper test was originally developed using data from military recruits and calibrated against realistic outdoor work rates representative of this population. Consequently, its fitted coefficients implicitly incorporate the average aerodynamic cost associated with typical running speeds for these subjects.

Within the speed range for which the Cooper test was intended—approximately 12–18 $\mathrm{km\,h^{-1}}$—this implicit treatment of air resistance yields reasonable estimates of aerobic capacity. Outside this range, however, the formulation becomes less robust: specifically at lower speeds the negative intercept leads to non-physical predictions as a consequence of negative RMR value.

Figure 4 provides a graphical comparison between the ACSM model (16) and the Cooper relation (18). For illustration, the figure also includes a nonlinear extension of the ACSM formulation in which the metabolic cost of aerodynamic drag is added explicitly, yielding

$$VO_2(v) = 3.33\,v + 3.5 + \frac{\rho C_d A}{2\,\eta_G^*\,m}\frac{60}{e_{O_2}}\left(\frac{v}{3.6}\right)^3, \tag{19}$$

where ρ denotes air density, $C_d A$ the effective aerodynamic drag area, m the body mass, and $\eta_{\mathrm{G,sat}}$ the quazi-gross efficiency according to Section 2.2. In Figure 4 it is seen that within reasonable human running speeds over 12 min duration, (19) agrees with the assymptote of the ACSM equation at low speeds and of the Cooper relation at high speeds.

3 A hybrid first-principles model for flat and incline running

In line with the literature review in Section 2.3, the metabolic energy expenditure during running may be expressed in the form

$$\mathrm{VO}_2 = m + k\,v + k_g\,\mathrm{grade}\,v, \tag{20}$$

where m, k, and k_g are empirical coefficients, v denotes running speed, and grade the treadmill incline. The contribution of the present work is the interpretation of these empirical parameters in terms of physically meaningful quantities associated with human locomotion.

3.1 Level running energetics

The constant term m is associated with the resting metabolic rate (RMR), also referred to as 1 MET, and is widely accepted to be approximately 3.5 mL kg^{-1} min^{-1} [17, 18].

The coefficient $k = 3.33$ mL/kg/min/(km/h) captures the energetic cost that increases approximately linearly with running speed. This contribution is commonly attributed to internal mechanical work, encompassing limb motion, muscle activation, elastic energy storage, and dissipative processes that dominate the metabolic cost of level running. Although more detailed biomechanical descriptions exist [19], they typically rely on numerous assumptions regarding running technique, muscle efficiency, and elastic energy utilization.

The corresponding mass-normalized equivalent mechanical power demand can consequently be expressed as

$$p_{\mathrm{eq}} = g\mu_{\mathrm{eq}}\,u. \tag{21}$$

In the present framework, mechanical power is related to metabolic power through an effective efficiency,

$$p_{\mathrm{met}} = \frac{g\mu_{\mathrm{eq}}\,u}{\eta^*}, \tag{22}$$

where η^* denotes an effective (quasi-gross) efficiency, cf. Section 2.2. From (22), the metabolic cost of transport can be identified as $C = g\mu_{\mathrm{eq}}/\eta^*$, cf. (6), which can be further converterd to running economy using (7)

$$\mathrm{RE} = 1000\frac{C}{e_{\mathrm{O}_2}} = 1000\frac{g\mu_{\mathrm{eq}}}{\eta^* e_{\mathrm{O}_2}}, \tag{23}$$

. Comparing to (20), we can now express

$$k = \frac{RE}{60} = \frac{g}{3.6}\,\frac{60}{e_{\mathrm{O}_2}}\,\frac{\mu_{\mathrm{eq}}}{\eta^*} = \frac{100g\mu_{\mathrm{eq}}}{6\eta^* e_{\mathrm{O}_2}}. \tag{24}$$

In the above formulation, η^* is introduced as an effective (quasi-gross) efficiency relating an equivalent mechanical power demand to metabolic power. Although gross efficiency (GE) is not uniquely defined for level running due to the absence of net external mechanical work, η^* provides a convenient and physically interpretable conversion factor within the present framework.

As discussed in Section 2.2, this effective efficiency is subsequently shown to coincide with the incline-saturated gross efficiency, $\eta_{G,\mathrm{sat}}$, which emerges naturally when mechanical work against gravity dominates the energetic cost of running. The central assumption of the present framework is therefore that the quasi-gross efficiency governing level running—where metabolic cost is dominated by internal dissipation—does not differ fundamentally from the saturated gross efficiency observed during steep uphill running, where external mechanical power output is well defined. The mathematical consequences of this assumption are further elaborated in Section 3.4.

3.2 Graded running energetics

The remaining term, $k_g\,\mathrm{grade}\,v$, admits a comparatively straightforward physical interpretation, as it originates from mechanical work performed against gravity.

When running at speed u ($\mathrm{m\,s^{-1}}$) on an incline with angle θ, the dominant additional energetic requirement is the mechanical work required to raise the body vertically. The corresponding mechanical power is

$$P_{\mathrm{mech,grav}} = mgu\sin\theta, \tag{25}$$

where m denotes body mass and $g \approx 9.81\ \mathrm{m\,s^{-2}}$ is the gravitational acceleration. Normalized by body mass, this yields a mechanical power per unit mass

$$p_{\mathrm{mech,grav}} = gu\sin\theta \quad [\mathrm{W\,kg^{-1}}]. \tag{26}$$

The metabolic power required to sustain this mechanical output is given by

$$p_{\mathrm{met,grav}} = \frac{p_{\mathrm{mech,grav}}}{\eta_I} = \frac{gu\sin\theta}{\eta_I}, \tag{27}$$

where η_I denotes the incremental efficiency associated with incline running, as defined in (11).

Expressing running speed in $\mathrm{km\,h^{-1}}$ as $v = 3.6\,u$, and using the small-angle approximation $\sin\theta \approx \mathrm{grade}$, the gravitational contribution to metabolic cost becomes proportional to $\mathrm{grade}\,v$, consistent with the empirical relation (20). Introducing the energetic equivalent of oxygen, $e_{\mathrm{O_2}} = 20.9\ \mathrm{J\,mL^{-1}}$, the corresponding oxygen uptake associated with gravitational work becomes

$$\mathrm{VO}_{2,\mathrm{grav}} = \frac{60g}{\eta_I\,e_{\mathrm{O_2}}}\,\frac{v}{3.6}\,\mathrm{grade}. \tag{28}$$

where the numerical factors arise from unit conversions between $\mathrm{m\,s^{-1}}$ and $\mathrm{km\,h^{-1}}$. Comparing (28) with (20), the coefficient multiplying $\mathrm{grade}\,v$ can be identified as

$$k_g = \frac{60g}{\eta_I\,e_{\mathrm{O_2}}}\,\frac{1}{3.6} = \frac{100g}{6\,\eta_I\,e_{\mathrm{O_2}}}. \tag{29}$$

Consistency with the empirical value $k_g \approx 0.15$ in the corresponding coefficient in the ACSM model yields $\eta_I \approx 0.52$, which is further discussed in Section 3.4.

3.3 Aerodynamic cost

Although aerodynamic drag is negligible for treadmill running and therefore absent in Margaria's original work [6] as well as in the ACSM treadmill model [7], it constitutes an important contribution to the metabolic cost of running under outdoor conditions, particularly at higher speeds. Accounting for aerodynamic drag is therefore essential for understanding how metabolic cost varies with speed and incline in outdoor running.

The aerodynamic drag force opposing forward motion may be expressed as

$$F_{\mathrm{air}} = \tfrac{1}{2}\,\rho\,C_d A\,u^2, \tag{30}$$

where $\rho \approx 1.2\ \mathrm{kg\,m^{-3}}$ denotes air density at sea level, C_dA is the effective drag area (product of drag coefficient and frontal area) in $\mathrm{m^2}$, and u is the running speed in $\mathrm{m\,s^{-1}}$.

Empirical studies of still-air resistance during locomotion indicate that the effective aerodynamic area during running is smaller than during walking, reflecting posture, limb motion, and dynamic body shape. Reported values for distance runners typically fall in the range $C_dA \approx 0.3$-$0.5\ \mathrm{m^2}$ for upright running, depending on body size, clothing, and running form [20]. As an example, at a running speed of $u = 5\ \mathrm{m\,s^{-1}}$ ($18\ \mathrm{km\,h^{-1}}$), this corresponds to an aerodynamic drag force on the order of 5-8 N, which may be compared to effective dissipative losses of magnitude $F_\mu = \mu_{\text{eq}} mg \approx 70$ N for representative values $\mu_{\text{eq}} = 0.1$ and $m = 70$ kg. The corresponding mechanical power required to overcome aerodynamic drag is given by

$$P_{\text{mech,air}} = F_{\text{air}}\, u = \tfrac{1}{2}\, \rho\, C_d A\, u^3, \tag{31}$$

where the cubic dependence on speed reflects the combined quadratic velocity dependence of the drag force and the additional factor of speed associated with power.

Expressed per unit body mass and converted to metabolic power using the quasi-gross efficiency $\eta_{G,\text{sat}}$, the corresponding mass-normalized metabolic power becomes

$$p_{\text{met,air}} = \frac{1}{\eta_{G,\text{sat}}} \frac{P_{\text{mech,air}}}{m} = \frac{1}{2\,\eta_{G,\text{sat}}} \frac{\rho\, C_d A}{m} u^3, \tag{32}$$

where m denotes body mass.

Finally, expressing the aerodynamic contribution in terms of oxygen uptake using the energetic equivalent of oxygen yields

$$\text{VO}_{2,\text{air}} = \frac{60}{e_{\text{O}_2}} p_{\text{met,air}} = \frac{30}{\eta_{G,\text{sat}}\, e_{\text{O}_2}} \frac{\rho\, C_d A}{m} \left(\frac{v}{3.6}\right)^3, \tag{33}$$

where $e_{\text{O}_2} \approx 20.9\ \text{J}\,\text{mL}^{-1}$ and the factor 3.6 accounts for the conversion between $\text{m}\,\text{s}^{-1}$ and $\text{km}\,\text{h}^{-1}$.

This cubic dependence on running speed explains why aerodynamic effects become increasingly relevant at higher outdoor running velocities.

3.4 Closed-form treadmill model with a special note on efficiency

A general closed-form model for energy expenditure in treadmill running may be obtained by combining the level-running and incline-related contributions derived above. Neglecting air resistance, the resulting model for treadmill running becomes

$$\begin{aligned} VO_2 &= \text{RMR} + \frac{60 g \mu_{\text{eq}}}{\eta^* e_{O_2}} u + \frac{60 g \sin\theta}{\eta_I e_{O_2}} u \\ &= \text{RMR} + \frac{100 g}{6 e_{O_2}} \left(\frac{\mu_{\text{eq}}}{\eta^*} + \frac{\sin\theta}{\eta_I} \right) v, \end{aligned} \tag{34}$$

where the second line follows from expressing the running speed in $\text{km}\,\text{h}^{-1}$, $v = 3.6u$. In (34), η^* and η_I denote the quasi-gross efficiency and the incline-referred incremental efficiency, respectively, as defined in Section 2.2.

Alternatively, (34) may be expressed in terms of running economy RE and grade $\approx \sin\theta$, yielding a form more similar in structure to the ACSM treadmill formula (16),

$$\begin{aligned} VO_2 &= \text{RMR} + RE \frac{v}{60} + \frac{100 g}{6\, \eta_I\, e_{\text{O}_2}} \,\text{grade}\, v \\ &= \text{RMR} + RE \frac{v}{60} + \frac{0.078}{\eta_I} \,\text{grade}\, v. \end{aligned} \tag{35}$$

Here v is expressed in $\text{km}\,\text{h}^{-1}$, $\text{RMR} \equiv VO_{2,\text{rest}} \approx 3.5$ in $\text{mL}\,\text{kg}^{-1}\,\text{min}^{-1}$, RE denotes running economy in $\text{mL}\,\text{kg}^{-1}\,\text{km}^{-1}$.

Comparing (35) to (16), running economy and incremental efficiency may be identified as

$$RE = 60 \cdot 3.33 = 200\ \text{mL}\,\text{kg}^{-1}\,\text{km}^{-1}, \qquad \eta_I = \frac{0.078}{0.15} \approx 0.52,$$

in order to comply with Margaria's original data and the ACSM treadmill model. Commenting on these values, $RE \approx 200\ \mathrm{mL\,kg^{-1}\,km^{-1}}$ agrees well with values reported for trained and recreational runners [21, 22, 23]. Similarly, the identified incremental incline efficiency, $\eta_I \approx 0.52$, is consistent with experimental observations for uphill locomotion [10].

At this stage, it is important to note that an efficiency of this magnitude would indeed appear unusually high if interpreted as a gross mechanical efficiency. However, η_I represents an incremental efficiency associated specifically with the additional mechanical work required to overcome gravity. Because the energetic cost of level running—including limb motion and internal mechanical work—is already accounted for in the baseline term, incremental efficiencies substantially exceeding gross efficiencies are both physiologically reasonable and experimentally observed.

The situation is fundamentally different for the efficiency of level running, denoted by η^*. As discussed in Section 2.2, this quantity is ill-defined in a strict mechanical sense due to the absence of net external work on level terrain. Nevertheless, within the present framework η^* serves as an effective conversion factor between internal mechanical dissipation and metabolic power, and must therefore be bounded by the intrinsic efficiency of muscular energy conversion. It is thus reasonable to assume that this effective (quasi-gross) efficiency does not differ significantly from the incline-saturated gross efficiency observed during steep uphill running, where mechanical power output is well defined. Retaining the general notation η^*, which is subsequently identified with the incline-saturated gross efficiency $\eta_{\mathrm{G,sat}}$, the mass-normalized metabolic power may be expressed as

$$\begin{aligned} p_{\mathrm{met}} &= \frac{e_{O_2}}{60}\,\mathrm{RMR} + \left(\frac{\mu_{eq}}{\eta^*} + \frac{\sin\theta}{\eta_I}\right) g\,u \\ &= \frac{e_{O_2}}{60}\,\mathrm{RMR} + \frac{RE\,e_{O_2}}{1000}\,u + \frac{g\sin\theta}{\eta_I}\,u. \end{aligned} \tag{36}$$

Based on the classical definition of gross efficiency, the only constructive mechanical work performed during treadmill running is that associated with vertical lifting of the center of mass. Gross efficiency may therefore be defined as

$$\eta_{\mathrm{gross}} = \frac{g\sin\theta\,u}{p_{\mathrm{met}}} = \frac{g\sin\theta\,u}{\frac{e_{O_2}}{60}\,\mathrm{RMR} + \left(\frac{\mu_{eq}}{\eta_{\mathrm{G,sat}}} + \frac{\sin\theta}{\eta_I}\right) g\,u}. \tag{37}$$

At this stage, three efficiency definitions appear explicitly in the formulation. This apparent complexity can be reduced by introducing a characteristic incline θ_I, hereafter referred to as the efficiency saturation angle, at which the gross efficiency is assumed to reach its saturated value, i.e. $\eta_{\mathrm{gross}}(\theta_I) = \eta_{G,\mathrm{sat}}$. Treating θ_I as an individual-specific parameter, substitution into (37) yields

$$\eta_{G,\mathrm{sat}} = \frac{gu\,(\sin\theta_I - \mu_{eq})}{\frac{e_{O_2}}{60}\,\mathrm{RMR} + \frac{gu\sin\theta_I}{\eta_I}}. \tag{38}$$

For sufficiently steep inclines, the gravity-related metabolic term dominates the resting metabolic rate,

$$\frac{gu\sin\theta_I}{\eta_I} \gg \frac{e_{O_2}}{60}\,\mathrm{RMR}, \tag{39}$$

in which case (38) simplifies to

$$\eta_{G,\mathrm{sat}} \approx \eta_I\left(1 - \frac{\mu_{\mathrm{eq}}}{\sin\theta_I}\right). \tag{40}$$

Alternatively, the internal losses may be expressed in terms of running economy rather than μ_{eq}. Retaining RMR and evaluating the expression along a $VO_{2,\max}$-limited trajectory gives

$$\eta_{G,\mathrm{sat}}(\theta_I;\eta_I) = \frac{g\sin\theta_I\,\left(VO_{2,\max} - \mathrm{RMR}\right)}{VO_{2,\max}\left(\frac{RE\,e_{O_2}}{1000} + \frac{g\sin\theta_I}{\eta_I}\right)}. \tag{41}$$

Neglecting RMR $\ll VO_{2,\mathrm{max}}$ further yields

$$\eta_{G,\mathrm{sat}}(\theta_I;\eta_I) = \frac{g\sin\theta_I}{\dfrac{e_{O_2}RE}{1000} + \dfrac{g\sin\theta_I}{\eta_I}} = \frac{1}{\dfrac{e_{O_2}RE}{1000\,g\,\sin\theta_I} + \dfrac{1}{\eta_I}}. \tag{42}$$

By substitution of RE in terms of μ_{eq}, (42) reduces to (40). Since $\sin\theta_I$ is bounded, it follows directly that (42) is bounded as well.

In the near-vertical limit $\theta \to 90^\circ$ (i.e. $\sin\theta \to 1$), this expression approaches

$$\eta_{\mathrm{G,sat}} \;\to\; \frac{1}{\dfrac{e_{O_2}RE}{1000\,g} + \dfrac{1}{\eta_I}} \approx \frac{1}{0.43 + \dfrac{1}{\eta_I}}, \tag{43}$$

where the numerical value 0.43 is obtained by inserting representative values $e_{O_2} \approx 20.9\ \mathrm{J\,mL^{-1}}$, $RE \approx 200\ \mathrm{mL\,kg^{-1}\,km^{-1}}$, and $g \approx 9.81\ \mathrm{m\,s^{-2}}$. This result implies that the gross efficiency is always lower than the corresponding incremental incline efficiency η_I, and is further constrained by realistic values of net efficiency, which according to experimental studies [10, 24] lie in the range $40\% < \eta_I < 60\%$.

Figure 5 illustrates the relationship between gross efficiency and incremental incline efficiency, with this experimentally relevant range highlighted. For realistic uphill grades on the order of 20–30%, the corresponding gross efficiency falls in the range $\eta_{\mathrm{gross}} \approx 22$–$28\%$, in good agreement with Minetti's observations of a gross mechanical efficiency of approximately 25% at steep inclines.

Motivated by this agreement, we return to expression (42) and consider the effective saturation behaviour at a representative grade of approximately 30%, where η_I has been experimentally shown to approach a plateau value. In this limit,

$$\eta_{\mathrm{G},30} \xrightarrow[\sin\theta\to 0.3]{} \frac{1}{\dfrac{e_{O_2}RE}{1000\,g\cdot 0.3} + \dfrac{1}{\eta_I}} \approx \frac{1}{1.43 + \dfrac{1}{\eta_I}}, \tag{44}$$

yielding $\eta_{\mathrm{G,sat}} \approx 30\%$ for $\eta_I \approx 52\%$, based on the parameter values implicit in the ACSM formulation.

This relationship provides a principled means of linking empirically observed efficiency plateaus to the gross-efficiency framework without introducing additional phenomenological parameters, and can be exploited to reduce the number of free parameters in the hybrid first-principles VO_2 model introduced in the following section.

It should be emphasized, however, that the ACSM model is known to overestimate VO_2 at steep inclines. This implies that the effective incremental incline efficiency under such conditions is likely closer to the lower end of the experimentally observed range, consistent with gross efficiencies clustering around 25% rather than 30%. Importantly, the precise numerical value of the saturated gross efficiency $\eta_{\mathrm{G,sat}}$ is not critical for the present reasoning. Rather, the key result is that a physically grounded relationship between the incremental incline efficiency η_I and the corresponding saturated gross efficiency $\eta_{\mathrm{G,sat}}$ has been established. Further, the treadmill running formula is interpreted in accepted dimensions of running economy RE, yielding a first-principles model where performance under various conditions can be linked to well-established physiological parameters rather than purely empirical constants.

3.5 An incline and speed-dependent model for outdoor running including aerodynamic component

Having defined the quazi-gross efficiency and its link to the incline referred incremental efficiency (44), it is now possible to insert this expression into (**??**). The energey expenditure including

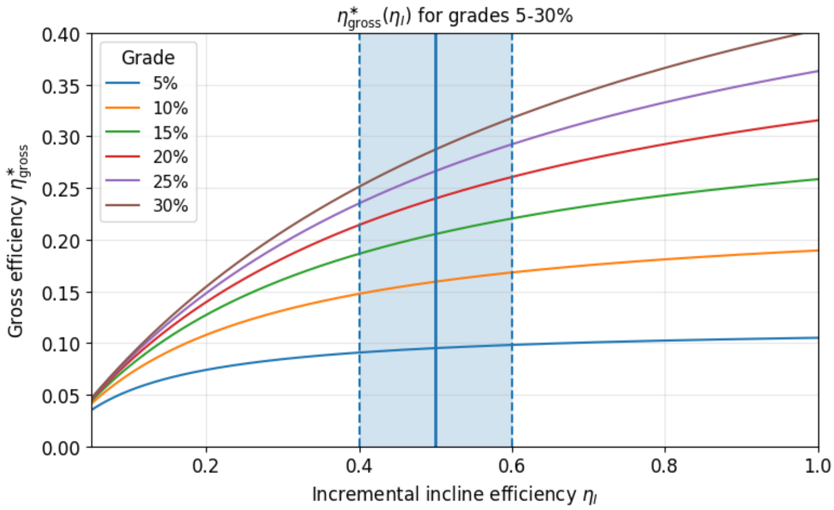


Figure 5: Gross efficiency versus incremental efficiency for different grades

aerodynamic drag can then be expressed

$$
\begin{aligned}
VO_2 &= \text{RMR} + RE\frac{v}{60} + \frac{g \cdot 60}{3.6\,\eta_I\, e_{\text{O}_2}}\,\text{grade}\, v + \frac{\rho C_d A}{2\,\eta_{\text{G,sat}}\, m\, e_{\text{O}_2}}\frac{60}{}\left(\frac{v}{3.6}\right)^3 \\
&= \text{RMR} + RE\frac{v}{60} + \frac{g \cdot 60}{3.6\,\eta_I\, e_{\text{O}_2}}\,\text{grade}\, v + \frac{\rho C_d A}{10m\, g}(RE + \frac{300g}{e_{\text{O}_2}\eta_I})\left(\frac{v}{3.6}\right)^3 \\
&= \text{RMR} + RE\frac{v}{60} + \frac{0.078}{\eta_I}\,\text{grade}\, v + 0.00028\frac{C_d A}{m}(RE + \frac{141}{\eta_I})v^3
\end{aligned}
\tag{45}
$$

.

In applied settings, the product of grade and speed is often expressed in terms of vertical ascent rate (VAR), measured as vertical meters per hour (vmph). Introducing this quantity into (45) yields an expression that can be expressed entirely in terms of physically interpretable parameters.

$$
\begin{aligned}
VO_2 &= \text{RMR} + RE\frac{v}{60} + \frac{g \cdot 60}{3.6\,\eta_I\, e_{\text{O}_2}}\,\text{grade}\, v + \frac{\rho C_d A}{2\,\eta_{\text{G,sat}}\, m\, e_{\text{O}_2}}\frac{60}{}\left(\frac{v}{3.6}\right)^3 \\
&= \text{RMR} + RE\frac{v}{60} + \frac{g}{60\, e_{\text{O}_2}}(\eta_{\text{G,sat}} - \frac{REe_{\text{O}_2}}{300g})\text{VAR} + \frac{\rho C_d A}{2\,\eta_{\text{G,sat}}\, m\, e_{\text{O}_2}}\frac{60}{}\left(\frac{v}{3.6}\right)^3 \\
&= \text{RMR} + 0.017RE\, v + 0.0078(\frac{1}{\eta_{\text{G,sat}}} - 0.0071RE)\text{VAR} + \frac{0.023}{\eta_{\text{G,sat}}\, m}v^3
\end{aligned}
\tag{46}
$$

where RMR is expressed in $\text{mL}\,\text{kg}^{-1}\,\text{min}^{-1}$, RE in $\text{mL}\,\text{kg}^{-1}\,\text{km}^{-1}$, v in $\text{km}\,\text{h}^{-1}$, and grade is dimensionless. The quantity $\text{VAR} \equiv 1000\, v\,\text{grade}$ denotes the vertical running speed in $\text{m}\,\text{h}^{-1}$. Furthermore, $g = 9.81\ \text{m}\,\text{s}^{-2}$, $e_{O_2} = 20.9\ \text{J}\,\text{mL}^{-1}$, ρ denotes air density in $\text{kg}\,\text{m}^{-3}$, $C_d A$ the effective aerodynamic drag area in m^2, and m the body mass in kg. In the final step, representative values $C_d A \approx 0.6\ \text{m}^2$ and $\rho \approx 1.2\ \text{kg}\,\text{m}^{-3}$ have been inserted.

It is now possible to convert to effective running power using (12),

$$P_{run} = RE\frac{e_{O_2}\eta_G^*}{60^2}v + g\cdot\eta_G^*\,(\frac{1}{\eta_G^*} - \frac{REe_{O_2}}{300g})\mathrm{VAR} + \frac{\eta_G^*\,\rho C_d A}{2\,\eta_G^*\,m}\left(\frac{v}{3.6}\right)^3$$
$$= RE\frac{e_{O_2}\eta_G^*}{60^2}v + \frac{1}{60^2}(g - \frac{\eta_G^* REe_{O_2}}{300})\mathrm{VAR} + \frac{\rho C_d A}{2\cdot 3.6^3 m}v^3 \tag{47}$$
$$= 0.0058\eta_G^* RE\, v + 0.00028(g - 0.070\eta_G^* RE)\mathrm{VAR} + \frac{0.0084}{m}v^3.$$

In the conversion to P_{run}, the resting metabolic rate (RMR) has been omitted, as it does not contribute to external mechanical power.

Equations (45)-(47) are cubic equations in the running speed v (km/h) and may be written on the canonical form

$$a\,v^3 + b\,v + c = 0, \tag{48}$$

where a, b, and c depend on grade and physiological parameters. Dividing Eq. (48) by $a \neq 0$ yields the depressed cubic

$$v^3 + p\,v + q = 0, \tag{49}$$

with

$$p = \frac{b}{a}, \qquad q = \frac{c}{a}. \tag{50}$$

The discriminant of Eq. (49) is

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3. \tag{51}$$

Cardano's formula yields the real solution

$$v = \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\Delta}}. \tag{52}$$

For the VO_2 formulation, collecting terms in v gives

$$p = \frac{\dfrac{RE}{60} + \dfrac{0.078}{\eta_I}\,\mathrm{grade}}{0.00028\,\dfrac{C_d A}{m}\left(RE + \dfrac{141}{\eta_I}\right)}, \qquad q = \frac{\mathrm{RMR} - \mathrm{VO_2}}{0.00028\,\dfrac{C_d A}{m}\left(RE + \dfrac{141}{\eta_I}\right)}. \tag{53}$$

For the present model, $p > 0$ for all physically admissible parameter values. Consequently, the discriminant Δ is strictly positive irrespective of the sign of q, and the cubic equation admits a unique real solution for the running speed v.

If the result is instead desired in terms of vertical ascent rate (VAR), which is often the case in applied settings, it can be obtained a posteriori from the solved running speed and the incline as

$$\mathrm{VAR} = 1000\,v\,\mathrm{grade}.$$

4 Discussion and model application

The purpose of this section is to discuss practical applications of the proposed framework for training analysis and performance monitoring.

As noted above, the concept of running power is largely inspired by cycling, where performance analysis is considerably more straightforward due to the relatively constant gross efficiency across typical riding conditions. In cycling, mechanical power therefore provides a direct and physically interpretable measure of exercise intensity, and higher power output generally implies higher performance.

In running, the situation is fundamentally different from cycling. Gross mechanical efficiency varies strongly with both speed and incline, necessitating a conceptually different approach. Commercial technologies that report running power are therefore necessarily based on proxy calculations rather than direct measurements of mechanically relevant work. Although such proxy-based running power metrics have been shown to correlate reasonably with physiological load [25], they cannot be interpreted or used in the same manner as cycling power. In particular, a higher reported running power does not necessarily indicate improved performance, but may instead reflect reduced running economy. This ambiguity in interpretation is likely one reason why running power has not achieved the same level of adoption as cycling power. Another reason is that, in level running, speed already correlates well with workload, reducing the practical need for an additional power metric.

The only situation in which running power can be meaningfully compared to cycling power is during uphill running, where gravitational work dominates the energetic cost. In this work, such uphill running is not treated as a separate special case, but is instead embedded within a unified energy-based framework that is consistent with level running. This provides a physically grounded basis for intensity control across running conditions, applicable to both indoor treadmill training and outdoor running in variable terrain, where speed alone becomes an increasingly ambiguous indicator of effort. As a practical application of this framework, Section 4.1 presents tables that enable the selection of metabolically equivalent workloads across different treadmill inclines. After addressing the comparatively idealized case of treadmill running, the more complex situation of outdoor running, where aerodynamic drag must be considered, is treated in Section 4.2.

4.1 Treadmill running

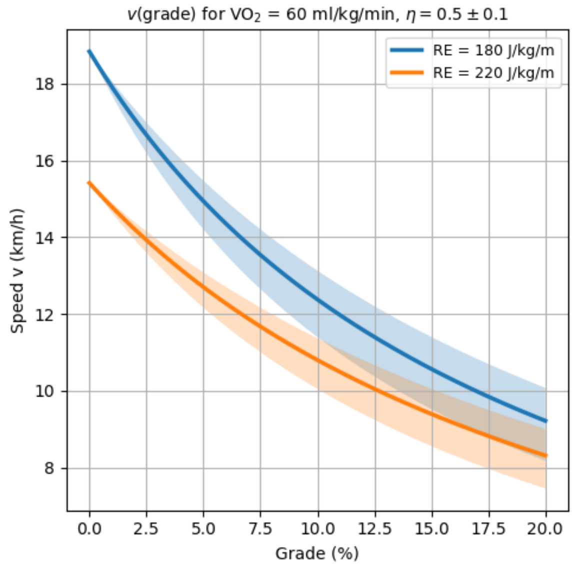
In the case of treadmill running, air resistance is absent and the metabolic energy expenditure is described by (**??**).

As evident from this relation, oxygen uptake VO_2 as a function of running speed and incline depends on the assumed values of running economy (RE) and the incline-referred incremental efficiency η_I. As an illustrative example, Table 4 presents VO_2 as a function of running speed for a range of treadmill inclines, expressed in degrees, for the representative parameter values $RE = 200\ \mathrm{mL\,kg^{-1}\,km^{-1}}$ and $\eta_I = 50\%$. These values are chosen to be consistent with those implicit in the ACSM treadmill running model, although slightly rounded.

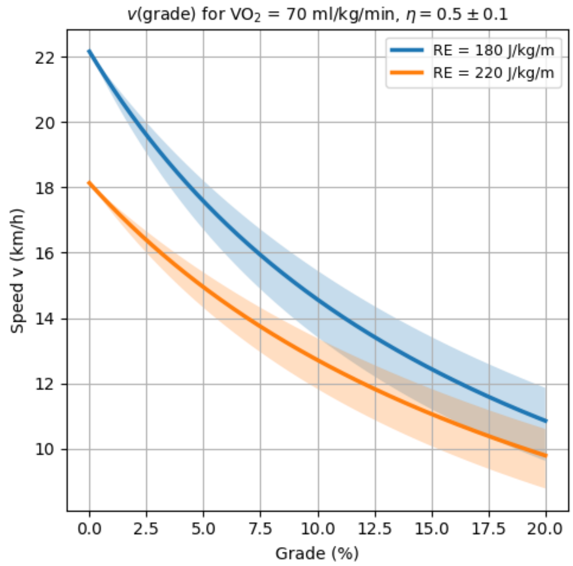
To facilitate practical speed selection at a prescribed metabolic intensity, Table 5 presents the inverse relationship, namely the equivalent running speed corresponding to a given VO_2 across a range of treadmill inclines.

The resulting speeds naturally depend on individual variations in running economy and incremental incline efficiency. To illustrate the sensitivity of the model predictions to deviations from the assumed parameter values, Figure 6 shows the dependence of running speed on variations in RE and η_I. At this point, it is also worth reflecting on the implications of assuming constant values of RE and η_I. In practice, both parameters are expected to vary to some extent with running conditions, which may partly explain why the ACSM model has been shown to exhibit limited precision under certain running scenarios [10, 11].

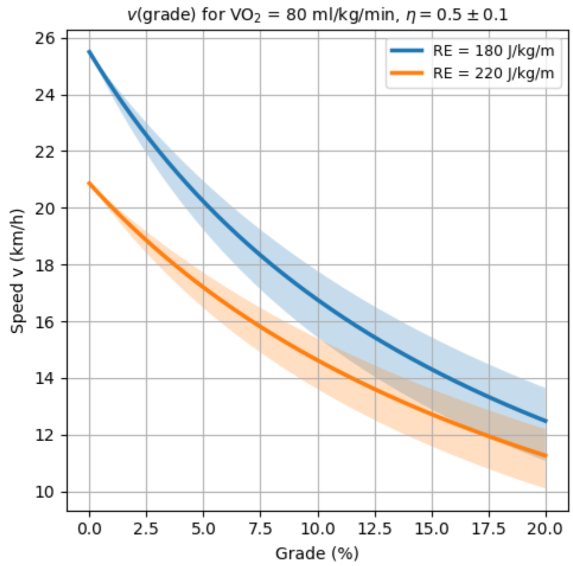
Figure 6 further illustrates that, for level running, performance variations are primarily determined by RE, whereas η_I, as expected, becomes increasingly influential as the incline increases. As the grade increases, differences in RE progressively lose importance. This effect is further accentuated by the fact that runners with lower RE in practice often exhibit higher η_I, since a larger fraction of internal losses is already incorporated into the measured RE.



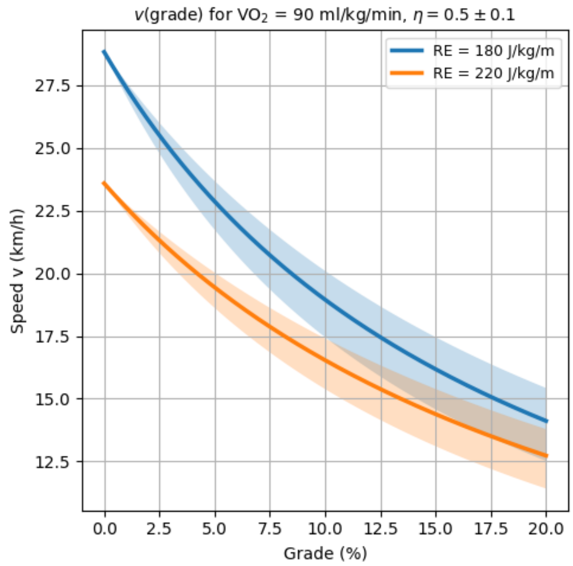
(a) $\mathrm{VO_2} = 60$ ml/kg/min



(b) $\mathrm{VO_2} = 70$ ml/kg/min



(c) $\mathrm{VO_2} = 80$ ml/kg/min



(d) $\mathrm{VO_2} = 90$ ml/kg/min

Figure 6: Speed as a function of grade for fixed $\mathrm{VO_2}$ levels. Solid lines show $\eta = 0.5$ and shaded regions indicate $\eta = 0.5 \pm 0.1$. Two curves are shown for $\mathrm{RE} = 180$ and $200\ \mathrm{J\,kg^{-1}\,m^{-1}}$.

4.2 Outdoor running

This section addresses the case of outdoor running, where aerodynamic drag contributes to the total metabolic energy expenditure. The relationship between running speed, incline, and oxygen uptake therefore differs from the treadmill case through the inclusion of air resistance.

Table 6 presents VO_2 versus speed and incline in the case of outdoor running where aerodynamic drag contributes to the metabolic cost. Note that in this case, the equivalent 3000 m time is very similar to the predictions according to Cooper's relation [5]. In outdoor running, incline is commonly reported as *grade* rather than as an angle. The two quantities are however related as

$$\mathrm{GRADE} = \tan\theta \approx \theta\frac{\pi}{180} \approx 1.75\,\theta, \tag{54}$$

where θ is given in degrees and GRADE is expressed in percent.

In practical outdoor applications, vertical work is often expressed in terms of *vertical ascent meters* (VAR), defined as

$$\mathrm{VAR} = 1000\, v \sin\theta \approx 10\, v\, \mathrm{GRADE}, \tag{55}$$

where v is the running speed in $\mathrm{km\,h^{-1}}$ and GRADE is expressed in percent. For convenience, Table 7 provides a direct conversion between running speed and vertical ascent rate for inclines ranging from 0 to 10°.

Table 4: VO_2 (ml/kg/min) according to developed Hybrid-First-Principle model applied on Treadmill running without air resistance (35). Assumed running economy $RE = 200$ ml/km/kg and incline-referred incremental efficiency η_I 50%. Predicted values exceeding $VO_2 = 100\ \mathrm{mL\,kg^{-1}\,min^{-1}}$ are shown in red, as they are considered to lie beyond typical human aerobic capacity.

v (km/h)	3000 m (mm:ss)	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
6	30:00	23	25	26	28	30	31	33	34	36	37	39
6,5	27:42	25	27	28	30	32	34	35	37	39	40	42
7	25:43	27	28	30	32	34	36	38	39	41	43	45
7,5	24:00	28	30	32	34	36	38	40	42	44	46	48
8	22:30	30	32	34	36	38	40	42	45	47	49	51
8,5	21:11	32	34	36	38	40	43	45	47	49	52	54
9	20:00	33	36	38	40	43	45	47	50	52	54	57
9,5	18:57	35	37	40	42	45	47	50	52	55	57	60
10	18:00	36	39	42	44	47	50	52	55	57	60	63
10,5	17:09	38	41	44	46	49	52	55	57	60	63	66
11	16:22	40	43	46	48	51	54	57	60	63	66	68
11,5	15:39	42	44	48	50	54	56	60	62	66	68	72
12	15:00	43	46	49	52	56	59	62	65	68	71	74
12,5	14:24	45	48	51	55	58	61	64	68	71	74	77
13	13:51	46	50	53	57	60	63	67	70	74	77	80
13,5	13:20	48	52	55	59	62	66	69	73	76	80	83
14	12:51	50	53	57	61	64	68	72	75	79	83	86
14,5	12:25	51	55	59	63	67	70	74	78	82	86	89
15	12:00	53	57	61	65	69	73	77	80	84	88	92
15,5	11:37	55	59	63	67	71	75	79	83	87	91	95
16	11:15	56	60	65	69	73	77	81	86	90	94	98
16,5	10:55	58	62	67	71	75	80	84	88	92	97	101
17	10:35	60	64	68	73	77	82	86	91	95	100	104
17,5	10:17	61	66	70	75	80	84	89	93	98	102	107
18	10:00	63	68	72	77	82	86	91	96	101	105	110
18,5	09:44	65	69	74	79	84	89	94	98	103	108	113
19	09:28	66	71	76	81	86	91	96	101	106	111	116
19,5	09:14	68	73	78	83	88	93	98	104	109	114	119
20	09:00	70	75	80	85	90	96	101	106	111	116	122
20,5	08:47	71	76	82	87	93	98	103	109	114	119	125
21	08:34	73	78	84	89	95	100	106	111	117	122	128
21,5	08:22	74	80	86	91	97	103	108	114	119	125	131
22	08:11	76	82	88	93	99	105	111	116	122	128	134
22,5	08:00	78	84	90	96	101	107	113	119	125	131	136
23	07:50	79	85	92	98	104	110	116	122	128	134	140
23,5	07:40	81	87	93	100	106	112	118	124	130	136	142
24	07:30	83	89	95	102	108	114	120	127	133	139	145
24,5	07:21	84	91	97	104	110	116	123	129	136	142	148
25	07:12	86	93	99	106	112	119	125	132	138	145	151

Table 5: Running speed v ($\mathrm{km\,h^{-1}}$) as a function of oxygen uptake VO_2 ($\mathrm{mL\,kg^{-1}\,min^{-1}}$) for different treadmill inclines. Assumed running economy $RE = 200$ ml/km/kg and η_I=50 % .

VO_2	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
30	8,0	7,4	6,9	6,4	6,1	5,7	5,4	5,1	4,9	4,7	4,5
32	8,6	7,9	7,4	6,9	6,5	6,1	5,8	5,5	5,3	5,0	4,8
34	9,2	8,5	7,9	7,4	7,0	6,6	6,2	5,9	5,6	5,4	5,1
36	9,8	9,0	8,4	7,9	7,4	7,0	6,6	6,3	6,0	5,7	5,5
38	10,4	9,6	8,9	8,4	7,9	7,4	7,0	6,7	6,4	6,1	5,8
40	11,0	10,2	9,5	8,9	8,3	7,9	7,4	7,1	6,7	6,4	6,1
42	11,6	10,7	10,0	9,3	8,8	8,3	7,9	7,5	7,1	6,8	6,5
44	12,2	11,3	10,5	9,8	9,2	8,7	8,3	7,8	7,5	7,1	6,8
46	12,8	11,8	11,0	10,3	9,7	9,2	8,7	8,2	7,8	7,5	7,2
48	13,4	12,4	11,5	10,8	10,2	9,6	9,1	8,6	8,2	7,8	7,5
50	14,0	12,9	12,1	11,3	10,6	10,0	9,5	9,0	8,6	8,2	7,8
52	14,6	13,5	12,6	11,8	11,1	10,5	9,9	9,4	8,9	8,5	8,2
54	15,2	14,0	13,1	12,3	11,5	10,9	10,3	9,8	9,3	8,9	8,5
56	15,8	14,6	13,6	12,7	12,0	11,3	10,7	10,2	9,7	9,2	8,8
58	16,4	15,2	14,1	13,2	12,4	11,7	11,1	10,6	10,1	9,6	9,2
60	17,0	15,7	14,7	13,7	12,9	12,2	11,5	10,9	10,4	9,9	9,5
62	17,6	16,3	15,2	14,2	13,4	12,6	11,9	11,3	10,8	10,3	9,9
64	18,2	16,8	15,7	14,7	13,8	13,0	12,3	11,7	11,2	10,7	10,2
66	18,8	17,4	16,2	15,2	14,3	13,5	12,8	12,1	11,5	11,0	10,5
68	19,4	17,9	16,7	15,7	14,7	13,9	13,2	12,5	11,9	11,4	10,9
70	20,0	18,5	17,2	16,1	15,2	14,3	13,6	12,9	12,3	11,7	11,2
72	20,6	19,1	17,8	16,6	15,6	14,8	14,0	13,3	12,6	12,1	11,5
74	21,2	19,6	18,3	17,1	16,1	15,2	14,4	13,7	13,0	12,4	11,9
76	21,8	20,2	18,8	17,6	16,6	15,6	14,8	14,0	13,4	12,8	12,2
78	22,4	20,7	19,3	18,1	17,0	16,1	15,2	14,4	13,7	13,1	12,5
80	23,0	21,3	19,8	18,6	17,5	16,5	15,6	14,8	14,1	13,5	12,9
82	23,6	21,8	20,4	19,1	17,9	16,9	16,0	15,2	14,5	13,8	13,2
84	24,2	22,4	20,9	19,5	18,4	17,3	16,4	15,6	14,9	14,2	13,6
86	24,8	22,9	21,4	20,0	18,8	17,8	16,8	16,0	15,2	14,5	13,9
88	25,4	23,5	21,9	20,5	19,3	18,2	17,2	16,4	15,6	14,9	14,2
90	26,0	24,1	22,4	21,0	19,8	18,6	17,7	16,8	16,0	15,2	14,6
92	26,6	24,6	22,9	21,5	20,2	19,1	18,1	17,1	16,3	15,6	14,9
94	27,1	25,2	23,5	22,0	20,7	19,5	18,5	17,5	16,7	15,9	15,2
96	27,8	25,7	24,0	22,5	21,1	19,9	18,9	17,9	17,1	16,3	15,6
98	28,3	26,3	24,5	22,9	21,6	20,4	19,3	18,3	17,4	16,6	15,9
100	29,0	26,8	25,0	23,4	22,0	20,8	19,7	18,7	17,8	17,0	16,3

Table 6: Oxygen uptake VO_2 ($\mathrm{mL\,kg^{-1}\,min^{-1}}$) predicted by the developed hybrid first-principles running model for outdoor running, including aerodynamic drag [cf. (45)]. The calculations assume an effective drag area $C_D A = 0.5\ \mathrm{m}^2$, a body mass of 75 kg, and a running economy $RE = 200\ \mathrm{mL\,kg^{-1}\,km^{-1}}$ and η_I=50%. Predicted values exceeding $VO_2 = 100\ \mathrm{mL\,kg^{-1}\,min^{-1}}$ are shown in red, as they are considered to lie beyond typical human aerobic capacity.

v (km/h)	3000 m (mm:ss)	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
6	30:00	24	25	27	28	30	32	33	35	36	38	39
6,5	27:42	25	27	29	31	32	34	36	37	39	41	42
7	25:43	27	29	31	33	34	36	38	40	42	44	45
7,5	24:00	29	31	33	35	37	39	41	43	45	47	48
8	22:30	31	33	35	37	39	41	43	45	47	49	52
8,5	21:11	32	35	37	39	41	44	46	48	50	52	55
9	20:00	34	37	39	41	44	46	48	51	53	55	58
9,5	18:57	36	38	41	43	46	48	51	53	56	58	61
10	18:00	38	40	43	46	48	51	53	56	59	61	64
10,5	17:09	40	42	45	48	51	53	56	59	61	64	67
11	16:22	41	44	47	50	53	56	59	61	64	67	70
11,5	15:39	43	46	49	52	55	58	61	64	67	70	73
12	15:00	45	48	51	54	58	61	64	67	70	73	76
12,5	14:24	47	50	53	57	60	63	67	70	73	76	80
13	13:51	49	52	56	59	62	66	69	73	76	79	83
13,5	13:20	51	54	58	61	65	68	72	75	79	82	86
14	12:51	53	56	60	64	67	71	75	78	82	86	89
14,5	12:25	55	58	62	66	70	74	77	81	85	89	92
15	12:00	56	60	64	68	72	76	80	84	88	92	96
15,5	11:37	58	63	67	71	75	79	83	87	91	95	99
16	11:15	60	65	69	73	77	81	86	90	94	98	102
16,5	10:55	62	67	71	75	80	84	88	93	97	101	106
17	10:35	64	69	73	78	82	87	91	96	100	104	109
17,5	10:17	67	71	76	80	85	89	94	99	103	108	112
18	10:00	69	73	78	83	88	92	97	102	106	111	116
18,5	09:44	71	76	80	85	90	95	100	105	109	114	119
19	09:28	73	78	83	88	93	98	103	108	113	118	122
19,5	09:14	75	80	85	90	95	101	106	111	116	121	126
20	09:00	77	82	88	93	98	103	109	114	119	124	129
20,5	08:47	79	85	90	96	101	106	112	117	122	128	133
21	08:34	82	87	93	98	104	109	115	120	126	131	136
21,5	08:22	84	90	95	101	106	112	118	123	129	134	140
22	08:11	86	92	98	104	109	115	121	127	132	138	144
22,5	08:00	89	94	100	106	112	118	124	130	136	141	147
23	07:50	91	97	103	109	115	121	127	133	139	145	151
23,5	07:40	93	99	106	112	118	124	130	136	142	149	155
24	07:30	96	102	108	115	121	127	133	140	146	152	158
24,5	07:21	98	105	111	117	124	130	137	143	149	156	162
25	07:12	101	107	114	120	127	133	140	146	153	159	166

Table 7: Vertical ascent (m) as a function of incline angle and running speed.

v (km/h)	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
grade (%)	1.75	3.49	5.23	6.98	8.72	10.5	12.2	13.9	15.6	17.4
6	105	209	314	418	523	627	731	835	938	1041
6.5	113	227	340	453	566	679	792	904	1016	1128
7	122	244	366	488	610	731	853	974	1094	1215
7.5	131	262	392	523	653	784	914	1043	1173	1302
8	140	279	418	558	697	836	974	1113	1251	1388
8.5	148	296	445	593	740	888	1035	1182	1329	1475
9	157	314	471	627	784	940	1096	1252	1407	1562
9.5	166	331	497	662	828	993	1157	1321	1485	1649
10	174	349	523	697	871	1045	1218	1391	1564	1736
10.5	183	366	549	732	915	1097	1279	1461	1642	1822
11	192	384	575	767	958	1149	1340	1530	1720	1909
11.5	201	401	602	802	1002	1201	1401	1600	1798	1996
12	209	419	628	837	1045	1254	1462	1669	1876	2083
12.5	218	436	654	872	1089	1306	1523	1739	1954	2170
13	227	453	680	906	1132	1358	1584	1808	2033	2256
13.5	235	471	706	941	1176	1410	1644	1878	2111	2343
14	244	488	732	976	1220	1463	1705	1947	2189	2430
14.5	253	506	758	1011	1263	1515	1766	2017	2267	2517
15	262	523	785	1046	1307	1567	1827	2087	2345	2603
15.5	270	541	811	1081	1350	1619	1888	2156	2424	2690
16	279	558	837	1116	1394	1672	1949	2226	2502	2777
16.5	288	576	863	1150	1437	1724	2010	2295	2580	2864
17	297	593	889	1185	1481	1776	2071	2365	2658	2951
17.5	305	610	915	1220	1524	1828	2132	2434	2736	3037
18	314	628	942	1255	1568	1881	2193	2504	2814	3124
18.5	323	645	968	1290	1612	1933	2253	2573	2893	3211
19	331	663	994	1325	1655	1985	2314	2643	2971	3298
19.5	340	680	1020	1360	1699	2037	2375	2713	3049	3384
20	349	698	1046	1394	1742	2090	2436	2782	3127	3471
20.5	358	715	1072	1429	1786	2142	2497	2852	3205	3558
21	366	733	1098	1464	1829	2194	2558	2921	3283	3645
21.5	375	750	1125	1499	1873	2246	2619	2991	3362	3732
22	384	767	1151	1534	1916	2298	2680	3060	3440	3818
22.5	392	785	1177	1569	1960	2351	2741	3130	3518	3905
23	401	802	1203	1604	2004	2403	2802	3199	3596	3992
23.5	410	820	1229	1638	2047	2455	2862	3269	3674	4079
24	419	837	1255	1673	2091	2507	2923	3338	3753	4165
24.5	427	855	1282	1708	2134	2560	2984	3408	3831	4252
25	436	872	1308	1743	2178	2612	3045	3478	3909	4339

5 Conclusions

This work has presented a unified and physically grounded interpretation of running energetics, formulated directly in terms of first-principles mechanics, physiological constants, and performance-related parameters. In contrast to purely empirical descriptions, the proposed framework explicitly distinguishes between the energetic cost of level running and the additional metabolic cost associated with graded locomotion through the introduction of an incline-referred incremental efficiency that is linked to an incline-saturated gross efficiency.

The incline-saturated gross efficiency is interpreted as a quasi–gross efficiency that enables conversion between mechanical work and metabolic load, irrespective of whether the mechanical work corresponds to external power production, as in uphill running, or to internal dissipation associated with limb motion, as in level running.

The incline-referred incremental efficiency is shown to approach a saturation value at steep inclines, consistent with experimentally reported values on the order of 50%. Importantly, this saturation behavior naturally implies a corresponding saturation of the gross mechanical efficiency at steep inclines, yielding values in the range of approximately 25–30%. These values are in good agreement with experimental observations and emerge directly from the model without the introduction of additional phenomenological assumptions.

Beyond the development of the first-principles framework, this work has reviewed and reconciled several established models of running energetics. It is shown explicitly that Margaria's classical results and the ACSM treadmill running model are compatible with the derived formulation under the assumptions of a representative running economy $RE \approx 200\ \mathrm{mL\,kg^{-1}\,km^{-1}}$ and an incline-referred incremental efficiency of approximately 52%. Furthermore, when standard quadratic air resistance is included with a realistic drag-area product $C_d A \approx 0.5\ \mathrm{m}^2$, the ACSM model is shown to be consistent with Cooper's outdoor running formulation, provided that performance is interpreted in terms of a saturated quasi–gross efficiency, GE_{sat}.

A key advantage of the proposed framework is that the quasi–gross efficiency introduced here provides a physically interpretable bridge between established running energetics and the more diffuse concept of "running power" that has emerged in recent years, largely driven by commercial wearable technologies. By relating metabolic power, mechanical work against gravity, and aerodynamic losses within a unified efficiency-based formulation, the model offers a principled means of interpreting externally reported running power metrics in physiological terms.

Finally, the practical applicability of the model is emphasized through the presentation of summary tables that enable straightforward use without recourse to the full analytical derivations. These tables are provided in two complementary forms: one for treadmill running, where aerodynamic drag is absent, and one for outdoor running, where air resistance is included. In the outdoor formulation, slope is expressed in terms of vertical ascent per unit distance, consistent with conventions commonly used in trail and mountain running. Together, these tabulated forms facilitate application of the model across laboratory, road, and off-road running contexts.

Future work will include further empirical studies of running economy and saturated efficiency values across a wider range of populations.

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